

Best response dynamics in waste-to-energy plants' price setting problem

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Abstract. This paper deals with the task of waste energy recovery and effective price setting of waste-to-energy plants. Its main contribution is the invention of a new approach to pricing of waste disposal in a particular waste management network. This novel method, which is based on a problem of finding the Nash equilibrium, takes into account the amounts of waste production of cities, capacities of waste-to-energy plants and their locations. The best response dynamics algorithm enables the computation of Nash equilibrium for a game of waste-to-energy plants in a normal form with numerous players and multiple strategies in comparison with other standard algorithms. The algorithm is able to find the Nash equilibrium for a sufficiently big step size between possible gate fees. The results of this work are applicable in the forecasting of prices of waste-to-energy plants' services for real waste management networks.

Keywords: waste management, waste-to-energy plant, gate fee, game theory, Nash equilibrium, best response.

Introduction

Waste management is an important and relevant topic that nowadays dynamically develops due to a significant increase in the world's attention to modern environmental problems. This accretion can be easily illustrated by an increasing number of various articles about waste management in recent years: according to the Web of Science search of the keyword "waste management" there are 1,955 articles on this topic in 2011 and 4,950 articles in 2018 [1]. The main issues of waste management are monitoring and regulation of collection, transportation, treatment and disposal of waste [2]. Waste management is closely connected to the concept of circular economy because one of their common goals is an effective treatment of solid waste.

The Circular Economy Package, which is a circular economy initiative adopted by the European Commission, sets up a series of goals which dwell in a decrease of the amount of solid waste that is being landfilled and an increase of its material and energy recovery. However, actual capacities of already built waste treatment facilities in some EU countries can be insufficient for that and new facilities should be built. Before this is done, an analysis of the feasibility of possible investments, waste transportation optimization and waste market modelling are needed.



Thus, various tools to support waste management planning have to be considered. They are usually built on optimization methods that are part of operations research techniques and used in design of sustainable supply chains. An article about application of optimization in designing of cost-effective waste management network in Thailand [3] and a study of optimization of routing in waste management network [4] can serve as examples. The review of the articles about applications of the above-mentioned means and assessment of the current state of the waste management studies are presented in [5]. In this article, we will focus on waste that is not suitable for material use; in this case, a highly effective waste-to-energy (WtE) plant technology should be considered.

At the Institute of Process Engineering of Brno University of Technology, a computational tool NERUDA [6] was developed supporting decision making in waste management. From the government's point of view, the results of its implementation are optimal and there are no complications. However, in case of solving a particular project applied to a certain locality, some of the parameters such as gate fees, which, for example, can correspond to a required return on investment, have to be fixed in order to maximize the revenues of WtE plants. That fact does not correspond to a real conflict of interests on a waste market and can be interpreted only as an analysis of a particular scenario.

Thus, the next logical step is an application of game theory that can by its nature provide a more realistic insight into market modelling and is a rational extension of existing approaches. By now, there is a certain lack of methods that use this area of mathematics to solve waste management problems. The majority of existing articles mainly use the basics of game theory. A brief literature review of the application of the game theory in waste management is presented at the end of the introduction, after the considered problem is described.

This article focuses on an identification of the price resulting state in an already built network (locations, productions and capacities are already given) that can be used in strategical planning (which plants should be built), forecasting of cash flow (how much will cities pay for waste disposal) and overall analysis of the waste management network (what capacities of the plants are sufficient and what locations are preferable).

In the game theory exist two basic approaches: cooperative and non-cooperative. The first one assumes that players can somehow agree on their decisions in order to increase their payoffs. However, in a price setting problem cooperation would mean the existence of the illegal collusion about price level that could damage the prosperity of consumers. Therefore, the non-cooperative approach, which describes a situation when each player pursues only his or her welfare without paying attention to profits of other players, will be used.

An approach to price setting problem used in the article mostly corresponds to [7]. WtE plants with specified capacities are located in an area and individually set their gate fees in order to maximize their income. Waste producers, who are represented by cities, are cooperatively minimizing their total costs with respect to the gate fees and transportation costs. The game occurs at the moment, when waste producers are trying to choose prices that will change demand for their prosperity. The primary purpose of the research is to find a stable set of gate fees such that none of WtE plants would like to change their gate fee. Particularly in this article, the well-known Nash equilibrium is considered to be such a stable point. The outcome can be used in solving further problems such as the interaction between cities or their usage of routes in waste management network to deliver waste to certain WtE plants.

In [8], the Stackelberg game of WtE incentive policy with a government in the leading role and dairy farms in the following role is discussed and solved. However, in the price setting problem of WtE plants, there is more than one player in the leading role, and a compromise between them is studied. The problem of finding the Nash equilibrium arises in [9] for effective funding of the recovery and recycling system and in [10] for invention of coordination strategies between participants of the market of the waste electrical and electronic equipment disposal in China. Nevertheless, in the above-mentioned articles, the considered payoff functions are continuous and differentiable; that assumption enables to analytically find the Nash equilibrium using first derivatives of the payoff

functions. In the model considered in this article, the payoff functions fail to meet the assumption of the continuity and differentiability. The evolutionary game theory approach performed in [11] cannot be used because the considered model is deterministic.

The contribution of this article is that a previous approach (based on an extensive search on small strategy spaces) to non-cooperative behavior of WtE plants in price setting problem [7] is improved by increasing cardinality of the sets of possible gate fees for which equilibrium can be found in adequate time. A novelty that enables such an improvement lies in using an algorithm based on the best response dynamics which will be useful even in case of many WtE plants in an arbitrary area. The sequential best response dynamics algorithm is based on the natural idea of non-cooperativity: every player independently sequentially maximizes his or her payoff function and this process, iteratively repeated, leads to an equilibrium state.

Materials and methods

All the concepts used in this section can be found in the standard book on game theory courses [12]. Considering the price setting a normal form game, where strategies of players are the gate fees, the main interest revolves around an analogy of stability when no player would like to change his or her decision. It will lead to the concept of a strategy profile of stable prices which corresponds to the Nash equilibrium and can be used in a further study of waste management problems. Thus, the main concern will be the computation of the Nash equilibrium.

Model description

Let $N = \{1, \dots, n\}$ be a set of WtE plants; w_1^c, \dots, w_n^c denotes their capacities and C_1^g, \dots, C_n^g denotes their strategy sets (sets of possible gate fees) with an element $c_j^g \in C_j^g, j \in N$. The set of producers is $M = \{1, \dots, m\}$. Their waste productions are w_1^p, \dots, w_m^p . Transportation costs are given by the matrix $[c_{i,j}^t]$, where $c_{i,j}^t$ represents the cost of waste transportation from the producer $i \in M$ to the plant $j \in N$. In the following expressions $x_{i,j}$ denotes the amount of waste sent by the producer $i \in M$ to the WtE plant $j \in N$ in tonnes. For each producer $j \in N$, the payoff function π_j is defined as $\pi_j(c_1^g, \dots, c_n^g) = \sum_{i \in M} c_j^g x_{i,j}^*$, where $x_{i,j}^* \in \{x_{i,j}^*: i \in M, j \in N\}$, such that

$$\{x_{i,j}^*: i \in M, j \in N\} = \arg \min_{x_{i,j}: i \in M, j \in N} \sum_{j \in N} \sum_{i \in M} (c_{i,j}^t + c_j^g) x_{i,j} \quad (1)$$

$$s. t. \sum_{i \in M} x_{i,j} \leq w_j^c, \forall j \in N, \quad (2)$$

$$\sum_{j \in N} x_{i,j} = w_i^p, \forall i \in M, \quad (3)$$

$$x_{i,j} \geq 0, \forall i \in M, \forall j \in N. \quad (4)$$

The previous equations describe cooperative minimization of total costs by cities and the fact that they have to dispose of all waste they produce and cannot exceed the capacities of WtE plants. A linear cost minimization problem, the solution of which is partially included in the players' payoff function, has to be solved in order to compute their payoff for a given strategy profile. However, the solution $\{x_{i,j}^*: i \in M, j \in N\}$ is not necessarily unique, thus a choosing rule, which will work equivalently for all of the players, should be established. There are two most common ways of choosing a particular solution: optimistic and pessimistic. The optimistic approach is the confidence of a player that if total costs of using his or her services equals the total costs of using services of another player for one of cities, then this city will always choose to work with him or her. Analogically, the pessimistic one describes the situation when a player expects payoff corresponding to the worst possible scenario. In this article the latter approach will be used.

By now two of three necessary milestones to define the normal form game of WtE plants have been established: the set of players $N = \{1, \dots, n\}$ and their payoff functions $\pi_j(c_1^g, \dots, c_n^g), j \in N$ have been defined. Only one question remains: what are the strategy sets of WtE plants?

The previously defined payoff functions are not differentiable or continuous. As a result, their derivatives cannot be described in order to analytically find the Nash equilibrium, and there is no special necessity to consider continuous strategy sets. This fact leads to an algorithmic way of finding the Nash equilibria. Moreover, a choice of discrete strategy spaces will help to avoid problems with the computation of maxima of payoff functions which occur in the case of continuous strategy sets. Therefore, in this article, the strategy sets of players will be described as discrete sets $C_j^g = \{kq, k \in N_0, k < K\}, j \in N$, where $q > 0$ is a step and K is a sufficiently large positive integer. In the next section, possible algorithms for finding the Nash Equilibrium will be discussed.

Algorithms

The first algorithm is a direct search of the Nash equilibrium. By definition of the Nash Equilibrium a strategy profile $c^{*,g} \in C_N^g = \prod_{j \in N} C_j^g$, such that $\pi_j(c_j^{*,g}, c_{-j}^{*,g}) \geq \pi_j(c_j^g, c_{-j}^{*,g}), \forall c_j^g \in C_j^g, \forall j \in N, c_{-j}^{*,g} = (c_i^{*,g})_{i \in N \setminus j}$, has to be found. In each step of the algorithm, an arbitrary strategy profile is chosen and checked whether the previous condition is satisfied for all possible strategies of each player. The problem arises when there are too many possible combinations: if 40 players with two strategies are considered, in the worst-case scenario it means checking 2^{40} possible combinations.

The idea of the second algorithm, which can be found in [13], dwells in the removal of dominated strategies. A strategy $c_j^{1,g} \in C_j^g$ is dominated by a strategy $c_j^{2,g} \in C_j^g$ if $\pi_j(c_j^{2,g}, c_{-j}^g) \geq \pi_j(c_j^{1,g}, c_{-j}^g), \forall c_{-j}^g \in C_{-j}^g$, and $\pi_j(c_j^{2,g}, c_{-j}^g) > \pi_j(c_j^{1,g}, c_{-j}^g)$ for some $c_{-j}^g \in C_{-j}^g$. Therefore, in the first step, all the dominated strategies of the first player are removed, and his or her strategy set is reduced to a set of undominated strategies. In the second step, the same approach is applied for the second player using the new reduced strategy set of the first player and so forth. If such an algorithm leads to a singleton, then this strategy profile is the Nash equilibrium. However, the issue remains: when there are 40 players and every player has at least two strategies, 2^{40} combinations have to be computed and checked in the first step of the algorithm.

The third algorithm is called sequential best response dynamics. Its detailed description can be found in the already mentioned text [13] or more specifically in [14]. The main idea of the algorithm is natural: a process starts at a given point and at each iteration every player chooses a strategy from his or her best response correspondence $B_j(c_{-j}^g) = \{c_j^g \in C_j^g: \pi_j(c_j^g, c_{-j}^g) \geq \pi_j(c_j'^g, c_{-j}^g), \forall c_j'^g \in C_j^g\}$. The new starting strategy profile for the next player is obtained from a chosen strategy of the previous player. If the algorithm converges to some strategy profile, then this profile is the Nash equilibrium. The advantage of the algorithm is apparent: at each step, only the maximum of a given player's payoff function has to be computed. In order to compute the maximum of the player's payoff function, all $K + 1$ points in his or her strategy set will be tried out with fixed prices of rivals. Thus, in one full iteration (the moment when all players would make their decision) $|N|(K + 1)$ linear programs have to be solved to find out a maximal payoff of each player. However, the main disadvantage of this algorithm is the fact that it can get stuck in a cycle. Figure 1 explains the main principles of the algorithm.

Comparing all the previous algorithms, it was decided to choose the sequential best response dynamics in order to solve the realistic exemplary problem due to its satisfactory computational complexity.

Case study

In this section results of applying sequential best response dynamics to the exemplary problem will be presented. The data that will be used in a case study does not describe any real region. However, the values of model parameters are realistic and were chosen in such way that they correspond to average figures valid for developed countries of the EU. The primary purpose of the case study is to show the functionality of the algorithm and non-triviality of the resulting equilibrium due to different operating conditions. The results of the computation of the Nash equilibria will be demonstrated starting with a large step between gate fees in strategy sets, and then it will be gradually reduced.

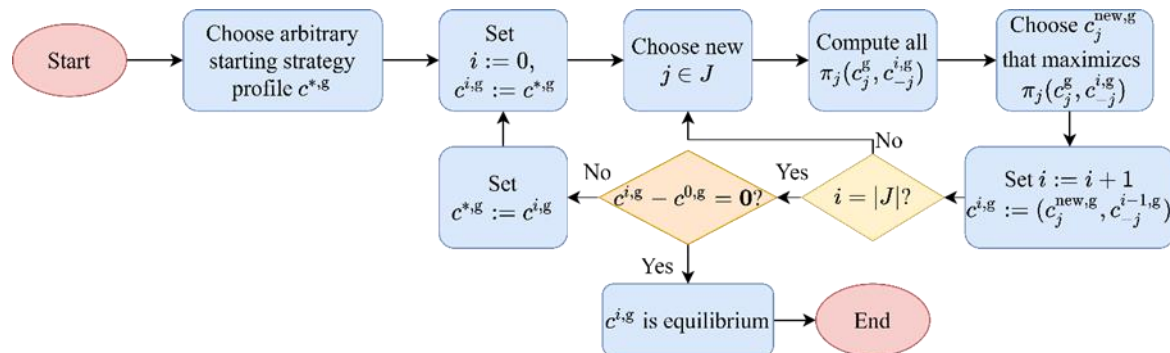


Figure 1. Flowchart of the algorithm based on the sequential best response dynamics.

Problem description

The exemplary problem is given by the data in table 1. In order to create an artificial upper bound in the strategy sets of players, the existence of a WtE plant with a fixed gate fee and capacity which will meet the total production of waste in the network will be considered (in real life that would mean already built foreign WtE plants, with price that cannot be impacted). Particularly, plant K with the capacity of 1,550 kt will have a fixed gate fee of 125 €. Productions and capacities are given for a period of one year.

Table 1. The network data of the exemplary problem.

Producer\WtE plant	Transportation costs (€/t)											Waste production (kt)
	A	B	C	D	E	F	G	H	I	J	K	
1	10	35	40	25	30	25	15	5	5	10	40	170
2	15	35	35	15	20	25	20	15	5	5	50	130
3	20	35	25	5	15	20	30	25	10	5	60	110
4	15	45	40	30	20	15	10	5	5	10	50	200
5	15	40	35	10	10	10	10	15	5	5	60	150
6	30	45	30	5	10	15	20	35	10	5	60	160
7	20	45	45	25	15	5	5	5	5	10	70	180
8	30	55	50	15	10	5	10	15	5	5	80	250
9	35	50	40	5	5	5	15	20	10	5	90	200
WtE plant capacity (kt)	50	75	275	150	125	250	100	175	200	200	1,550	

As best response dynamics could not answer the question about the uniqueness of equilibria, three starting strategy profiles (lower bound, middle of strategy sets and upper bound) will be tried out for each chosen step in order to find out if the algorithm converges to the same result. Obviously, it is not sufficient to make a general conclusion about the uniqueness of the found equilibria. Analogically, if the algorithm gets stuck in a cycle for all of the considered starting strategy profiles, the existence of

equilibria for the given step cannot be disproved because a more complex and detailed analysis of the situation is needed.

Results and discussion

The results of the algorithm execution are presented in table 2. The step size of 22.5 €/t is the smallest for which the algorithm converges and after reducing it, a cycle begins to occur for each starting point. Thus, the equilibrium for that step (the penultimate row in table 2) will be considered as the most precise solution of the price setting problem. From the stable gate fees and information about used capacities an overall analysis of the situation can be made. The rationality of WtE plants guarantees that plant K with a fixed gate fee will not be used by waste producers. WtE plants B and C with inconvenient locations (represented by high transportation costs) are still competitive; however, their gate fees are lower than the gate fees of their rivals and C is the only plant with 18 % of capacity left unused. All WtE plants except B and C have the same gate fee, which points out similar operating conditions.

Table 2. The results of the algorithm execution.

Step size [€]	Starting point [€]	The highest possible price [€]	Time of execution [s]	Gate fee [€]									
				A	B	C	D	E	F	G	H	I	J
100	0	400	1.72	100	100	100	100	100	100	100	100	100	100
100	200	400	2.36	100	100	100	100	100	100	100	100	100	100
100	400	400	2.99	100	100	100	100	100	100	100	100	100	100
50	0	300	1.41	100	100	100	150	100	150	150	150	150	150
50	150	300	2.78	100	100	100	150	100	150	150	150	150	150
50	300	300	4.57	100	100	100	150	100	150	150	150	150	150
25	0	300	2.6	125	100	125	125	125	150	150	150	150	150
25	150	300	5.24	125	100	125	125	125	150	150	150	150	150
25	300	300	8.58	125	100	125	125	125	150	150	150	150	150
22.5	0	315	2.89	135	90	112.5	135	135	135	135	135	135	135
22.5	157.5	315	6.98	135	90	112.5	135	135	135	135	135	135	135
22.5	315	315	12.1	135	90	112.5	135	135	135	135	135	135	135
Capacity used for step size 22.5 €/t [%]				100	100	82	100	100	100	100	100	100	100

In the considered exemplary problem the difference between the total waste production and the total capacity of the all WtE plants with the non-fixed price is rather small, this is the reason why percentages of the used capacities are so high and the operation of the whole network will be sustainable. However, during strategic planning in the big size waste management network, problems with a small use of capacity could occur. This fact will lead to some requirements: for example, considered configuration of WtE plants will be built only if more than 60 % of each plant's capacity will be used. At this point a further complication arises: elimination of the WtE plant with the smallest use of capacity is not always optimal (from the customers' point of view) and could lead to an increase in total waste producers' expenses. Therefore, all possible eliminations should be compared and the optimal one should be selected. If the problem occurs again, this process should be repeated iteratively. Such an approach could enable a stable compromise between the prosperity of producers and WtE plants.

Observing the results, some general statements about best response dynamics application and the found Nash equilibria of the considered price setting problem can be made: for sufficiently large steps the algorithm converges to the Nash equilibrium, which corresponds to the lowest possible gate fees (except zero); with reducing the step size, for some WtE plants it becomes more profitable to deviate

to some greater gate fees, but the algorithm is still able to find the Nash equilibrium; from a particular step size, the algorithm gets stuck in a cycle with at least two WtE plants constantly changing their gate fees and there exists a possibility, that a change of gate fee is always profitable for a small step size and no stable point exists.

In this article, only the basic sequential best response dynamics algorithm with exhaustive search method optimization on discrete sets has been used. Therefore, there are ways of improvement of the considered approach. Instead of exhaustive search method, more rational ways of finding the maxima of payoff functions using bilevel optimization techniques, which can gradually reduce the time of computation, can be used. Moreover, such techniques could enable us to find optimality conditions of players from which the equilibrium can be analytically found.

Ways of choosing the best solution should be discussed in further studies of this issue. Analogically to approach used in this article, the solution found for the smallest step size, for which the algorithm still converges, could be considered the most precise. Thus, the use of heuristics in best response dynamics to reduce the step for which algorithm is able to find the equilibrium should also be considered: players can admit a decrease in their payoffs in order to avoid cycle or can be satisfied with points in a neighborhood of the actual Nash equilibrium.

The further development of the research in this area will focus on the application of an invented method on real data and detailed analysis of results based on actual operating conditions in a region.

Conclusion

In this article, the game theory has been applied to the process of setting gate fees by WtE plants in order to show possible applications of this area of mathematics in the analysis of the sustainable waste management networks. The mathematical model of conflict between WtE plants has been described and the corresponding normal form game has been defined. Moreover, the aim to find the Nash equilibrium of this normal form game has been set in order to get an outline of the possible stable outcome of the mentioned process. After that, algorithms for finding the Nash equilibria were discussed and compared. The sequential best response dynamics has been chosen as an appropriate algorithm due to its computational complexity: it is able to find the Nash equilibrium of the normal form game with many players and large strategy sets in adequate time. Results of applying best response dynamics to the problem with realistic waste management data have been presented together with the brief analysis of the exemplary waste management network based on the found equilibria. Moreover, observation of the results has enabled the establishment of ideas about some consistent pattern of the Nash Equilibria values for specific step sizes in strategy sets. This idea has been described in three statements which are part of subsection 3.2. Future research on that topic might be devoted to mathematical proofs of those statements, implementing rational bounds in the strategy sets, searching for conditions of existence and uniqueness of the Nash equilibria for the WtE plants' games in a normal form and improving the algorithm by using heuristics or bilevel programming methods. The main contribution of the invented approach is that due to its speed it enables us to effectively set gate fee in an arbitrary waste management network and can be used in the iterative process of design of sustainable waste management networks described in subsection 3.2. This could lead to the network design in which expenses of waste producers are minimal, and all plants use most of their capacities with a sustainable gate fee.

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