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LETTER

Variation of a classical fingerprint of ideal memristor

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SUMMARY

Gradual disappearance of hysteresis with increasing frequency of the exciting signal is considered a classical fingerprint of general memristive systems. The paper analyzes a special version of this fingerprint, when the value of the charge delivered within the half-period remains constant while the frequency of the sinusoidal current is increasing. Under these conditions, the area of the pinched hysteresis loop of the ideal memristor increases with the square of the frequency. Breaking the rules of this fingerprint indicates reliably that the element analyzed is not an ideal memristor. Copyright © 2015 The Authors International Journal of Circuit Theory and Applications Published by John Wiley & Sons, Ltd.

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1. INTRODUCTION

The fact that the area of the pinched hysteresis loop of the memristive system disappears with increasing frequency of periodical excitation follows from the proof given by Chua and Kang [1] in 1976. It is assumed in [1] that the driving signal remains bounded in values within the process of increasing its frequency.

Because the ideal memristor defined in [2] (hereinafter the memristor) is a special case of a memristive system, the aforementioned fingerprint also applies to it. Consider a memristor whose memristance R_M is controlled by the charge q modeled by state-dependent Ohm's law in the form

$$v = R_M(q)i. \tag{1}$$

Its v–i characteristic is not unambiguous because the instantaneous memristance depends on the integral of current or, in other words, on the history of excitation. The corresponding curve forms the pinched hysteresis loops, along which the operating point repeatedly comes back to the v–i origin if voltage and current cross the zero levels, as obvious from Figure 1(b). The loop area can be considered a measure of the memory effect of the memristor [3].

The so-called constitutive relation (CR) is a native characteristic of the element, predicating about the element itself, being independent of both the excitation and the neighboring network [4]. For the memristor, the CR is the unambiguous charge q (integral of current) versus flux φ (integral of voltage) relationship. The CR in the form $\varphi(q)$ is the first integral of the memristance with respect to

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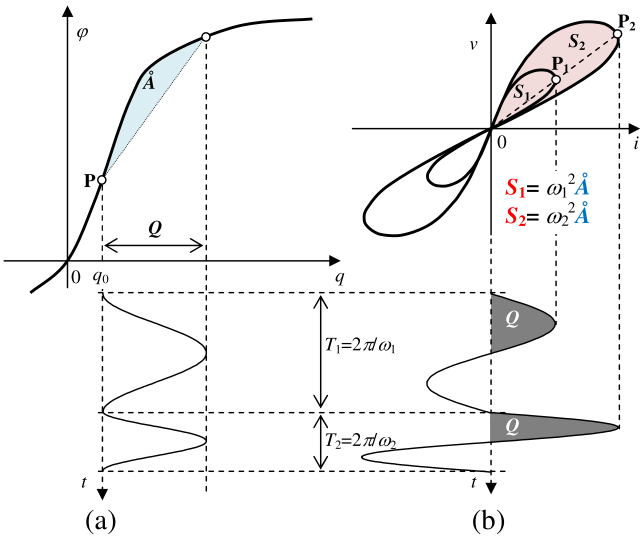

Figure 1. (a) Constitutive relation $\varphi(q)$ and the charge waveform and (b) pinched hysteresis loops drawn by two different periods of the current, which convey the same charge.

the charge [5]. Then, the instantaneous memristance is given by the slope of the CR curve at the current operating point; thus, $R_M(q) = d\varphi/dq$. The initial charge q_0 involves the entire history of the current action until the time instant when the defined excitation starts to operate, and it determines the initial operating point P on the CR curve, Figure 1(a).

Assume that the memristor current is

$$i(t) = I_{MAX} \sin(\omega t), \tag{2}$$

where I_{MAX} and ω are the amplitude and the angular frequency, respectively. The charge passed through the memristor within the half-period $T/2$ is as follows:

$$Q = \int_0^{T/2} I_{MAX} \sin(\omega t) dt = \frac{2I_{MAX}}{\omega}. \tag{3}$$

The operating point P is moved along the CR from the position that corresponds to the charge q_0 to another position that corresponds to the charge $q_0 + Q$. It is proved in [6] that the following formula for the loop area holds for the sinusoidal excitation

$$S = \omega^2 \mathring{A}, \tag{4}$$

where $\mathring{A}$ is a part of the so-called *action* of the memristor, being represented by the area in the φ–q plane according to Figure 1(a). It follows from (4) that, for example, if $T_1 = 2T_2$ in Figure 1, then the ratio of the areas S_1 and S_2 will be 1:4.

The aforementioned state of the art can be utilized for alternative statement of the classical fingerprint for the case of the memristor driven by a source of charge. It will be shown that, in contrast to the classical case when the driving voltage or current remains bounded in values, the area of the pinched hysteresis loop will surprisingly be increasing with the square of the frequency.

2. ALTERNATIVE STATEMENT OF CLASSICAL FINGERPRINT

The authentic statement of the classical fingerprint of diminishing the hysteresis postulates that the periodical driving signal remains within predetermined bounds when its frequency is increasing [1]. The development of the hysteresis will be studied hereafter under different conditions of the charge

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excitation: The source modifies the frequency of the sinusoidal current and concurrently also its amplitude such that an *equal amount of the charge* passes through the memristor within each half-period independently of the frequency. It is obvious from (3) that the source must maintain a linear relationship between the frequency and the amplitude and that the original demand for the driving signal to be bounded is not applicable anymore.

Figure 1(b) illustrates excitation in the form of two successive periods of sinusoidal current of different lengths of the periods and different amplitudes, whereas the charges delivered within the first and third half-periods are identical. While two different hysteresis loops are drawn in the v–i plane, the operating point follows the same curve in the φ–q plane (see the CR curve in Figure 1(a)) within both repeating periods. It is due to the equality of charges conveyed within each half-period. The only difference is in the speed of the motion, which is given by the length of the current period. If both the frequency and the amplitude of the sinusoidal excitation are modified such that the charge q remains within the interval $< q_0,\ q_0 + Q >$ with constant Q, the area $\mathring{A}$ remains also constant, and, according to (4), the loop area increases with the square of the frequency. Then, the alternative statement of the fingerprint of the frequency-dependent hysteresis is as follows:

If the sinusoidal current source delivers the same amount of the charge into the ideal memristor within each half-period independently of the frequency, then the area of the v–i pinched hysteresis loop increases with the square of the frequency.

The identical amount of the charge within each half-period represents a requirement of zero mean value of the current. Then, it is guaranteed that the operating point will return to the same starting position on the CR curve after each repeating period.

The current excitation (2) can be rewritten in the form

$$i(\alpha) = \frac{\omega Q}{2} \sin \alpha, \tag{5}$$

where $\alpha = \omega t$. Then, the charge as a function of the angle α is

$$q(\alpha) = \int_0^{\alpha} \frac{\omega Q}{2} \sin \alpha' \frac{d\alpha'}{\omega} = \frac{Q}{2}(1 - \cos \alpha). \tag{6}$$

Then, according to (5) and (1), the memristor current $i(\alpha)$ and voltage $v(\alpha)$ are directly proportional to the driving frequency. Such points in the collection of v–i hysteresis loops according to Figure 1(b), which correspond to the same angle $\alpha = \omega t$, are lying on a ray that starts from the v–i origin, whereas the distances of these points from the origin are in the same relationships as the corresponding frequencies (see points P_1 and P_2 in Figure 1(b)). From the mathematical point of view, the pinched hysteresis loops are *homothetic* [7] with the homothetic center at the v–i origin, which implicates the following rule of homothety:

If the source of sinusoidal current delivers into the ideal memristor the same amount of charge during each half-period independently of the frequency, then the v–i pinched hysteresis loop for the frequency ω_1 *is a homothetic entity with respect to the loop for the frequency* ω_2*, with the homothetic center at the v–i origin and the ratio of homothety* $\kappa = \omega_1/\omega_2$.

As a consequence of the homothety, the hysteresis loops preserve their shape when varying the frequency with fixed Q, only changing their size [7].

Note that the aforementioned fingerprint and the inferential rule of homothety hold for the ideal memristors. Nevertheless, this does not mean that they cannot also hold for some of the memristive systems that are not ideal memristors. In either case, breaking the rules of the fingerprint indicates reliably that the element analyzed is not an ideal memristor.

3. SIMULATION

The aforementioned fingerprint and the corresponding rule of homothety are confirmed via the following simulations, based on a model of memristive system that is a thought modification of the well-known 'HP (Hewlett-Packard) memristor' [8]. Such a modification consists in the dependence of the memristance not only on the charge q but also on the current i according to the formula

$$R_M(q,i) = R_M(q_0, i_0) + k(q - q_0)e^{A|i|}, \tag{7}$$

where k and A are constants and q_0 and i_0 are the initial charge and current, respectively. For $A=0$, the aforementioned model is simplified into the well-known model of the HP memristor with linear dopant drift [6], that is, model of ideal memristor. Note that (7) does not correspond to any model of the currently known physically existing device. The exponential term was added to (7) in order to have a possibility to pass smoothly between an ideal memristor and a more general memristive system via modifying only one parameter A.

The memristive element (7) is driven by a sinusoidal current, delivering a charge of 3 mC within each half-period independently of the frequency. The corresponding pinched hysteresis loops for the frequencies from 2 to 40 Hz are presented in Figure 2. The parameters of the element are selected as follows: $k=9978\,\Omega/\text{C}$, A is 0, 5, and -10, the initial charge $q_0=0$, the initial current $i_0=0$, and the initial memristance $R_M(q_0, i_0)=11\,\Omega$.

Figure 2(a) shows the hysteresis loop of the element (7) for $A=0$, thus for the ideal memristor fulfilling the fingerprint and the rule of homothety. It holds for every time instant that the ratios of the voltages (currents) are 2:4:6: … :40 for the frequencies 2:4:6: … and 40 Hz, such that the curve preserves its shape and the loop area increases with the square of the frequency. The loops are homothetic with respect to the v–i origin.

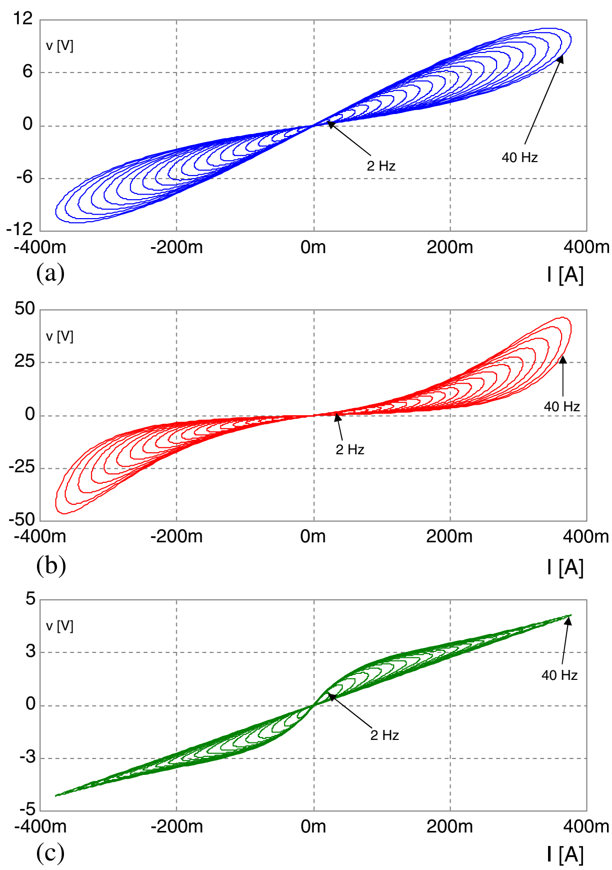

Figure 2. Pinched hysteresis loops of the model from Eq. (7) for increasing frequency from 2 to 40 Hz; the charge conveyed within the half-period remains constant (3 mC). For ideal memristor, (a) the loops preserve their shape; for memristive systems (b), (c) the shapes are modified with changing frequency.

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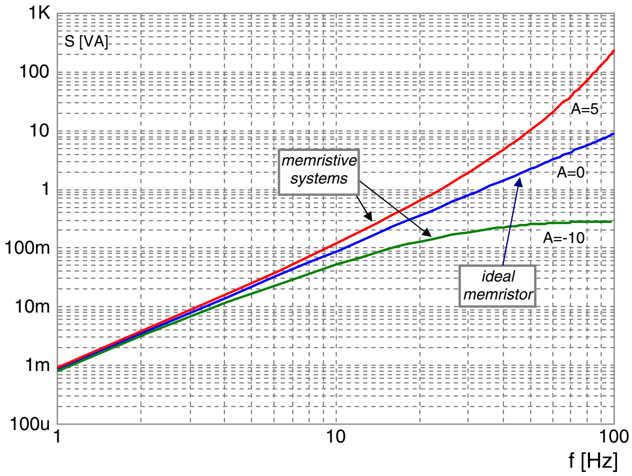

Figure 3. The loop area versus the frequency of the model from Eq. (7) for a constant charge conveyed within a half-period. The law is parabolic (ω^2) for ideal memristor.

The pinched hysteresis loops in Figures 2(b) and 2(c) cannot be loops of an ideal memristor because they change their shape with changing frequency and the loop area versus frequency does not copy the square law. The actual area versus frequency dependence relations are shown in Figure 3.

It follows from the requirement of the parabolic $S(\omega)$ law for the ideal memristor that the ratio of the loop areas for arbitrary frequencies ω_1 and ω_2 must be

$$\frac{S(\omega_1)}{S(\omega_2)} = \left(\frac{\omega_1}{\omega_2}\right)^2 = \kappa^2. \tag{8}$$

This conclusion is in agreement with the observation in [7] that the ratio of the areas of two mutually homothetic entities is equal to the square of the ratio of homothety.

As is obvious from Figure 1(a), $\mathring{A}$ represents a contribution of the CR nonlinearity to the total value of action during the movement of the operating point within one half-period. When increasing k times both the frequency and the amplitude, the swing of the movement of the operating point along the CR in Figure 1(a) remains unchanged, and the value of $\mathring{A}$ remains preserved. From this point of view, Eq. (8) is a logical consequence of formula (4).

4. CONCLUSIONS

The homothety of pinched hysteresis loops of ideal memristors under their excitation via sinusoidal current preserving the charge is a direct consequence of a novel fingerprint of the quadratic loop area versus frequency relationship. Breaking the rules of the fingerprint or the consequent rule of homothety indicates reliably that the memristive element under consideration is not an ideal memristor. The evaluation of such a discrepancy can be performed at a glance by inspection of the corresponding set of v–i characteristics. On the other hand, the mechanism of the novel fingerprint is more evident in the light of the rule of homothety: The areas of homothetic entities depend on the square of the ratio of homothety or on the square of the ratio of frequencies (8).

As follows from the procedure of its derivation, the fingerprint holds for arbitrary ideal memristors, thus for memristors with arbitrary possible memristance versus charge map $R_M(q)$, even if they can produce pinched hysteresis loops of various shapes [9]. Applying the duality principle, one can deduce that the aforementioned fingerprint also applies to ideal memristors driven by a sinusoidal voltage source that delivers an identical flux φ within each half-period independently of the frequency.

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