Williams expansion-based approximation of the stress field in an Al 2024 body with a crack from optical measurements

S. Seitl, L. Malíková
Brno University of Technology, Faculty of Civil Engineering, Institute of Structural Mechanics, Veveří 331/95, 602 00 Brno, Czech Republic and
Academy of Sciences of the Czech Republic, Institute of Physics of Materials, v. v. i., Žižkova 22, 616 62 Brno, Czech Republic
seitl@ipm.cz, http://orcid.org/0000-0002-4953-4324
malikova.l@fce.vutbr.cz, http://orcid.org/0000-0001-5868-5717

J. Sobek, P. Frantík
Brno University of Technology, Faculty of Civil Engineering, Institute of Structural Mechanics, Veveří 331/95, 602 00 Brno, Czech Republic
Sobek.j@fce.vutbr.cz, http://orcid.org/0000-0003-4215-1029
frantik.p@fce.vutbr.cz,

P. Lopez-Crespo
Department of Civil and Materials Engineering, University of Malaga, C/Dr Ortiz Ramos s/n, 29071 Malaga, Spain
plopezcrespo@uma.es

ABSTRACT. A study on the approximation of the stress field in the vicinity of crack tip in a compact tension specimen made from Al 2024-T351 is presented. Crack tip stress tensor components are expressed using the linear elastic fracture mechanics (LEFM) theory in this work, more precisely via its multi-parameter formulation, i.e. by Williams power series (WPS). Determination of coefficients of terms of this series is performed using a least squares-based regression technique known as over deterministic method (ODM) for which displacements data obtained experimentally via optical measurements are taken as inputs. The stress fields reconstructed based on the displacement data obtained experimentally by means of optical measurements are verified by means of the stress field approximations derived for the normalized CT specimen via hybrid elements.

KEYWORDS. Crack tip fields; Optical measurements; DIC; Williams power series; Higher-order terms; Stress field reconstruction; Multi-parameter approximation.

OPEN ACCESS

Citation: Seitl, S., Malíková, L., Sobek, J., Frantík, P., Lopez-Crespo P., Williams expansion-based approximation of the stress field in a Al2024 body with a crack from optical measurements, Frattura ed Integrità Strutturale, 41 (2017) 323-331.

Received: 28.02.2017
Accepted: 03.05.2017
Published: 01.07.2017

INTRODUCTION

Characterization of crack tip stresses has been an area of active research for many decades; see short state-of-art in [1, 2] presenting an over-deterministic least squares technique to evaluate the mixed-mode multi-parameter stress field by photo- elasticity underlining the fact that the use of a multi-parametric representation is not just for academic curiosity but a necessity in some cases of engineering interest. This fact is also underlined by Ayatollahi et al. in [3] who used displacement fields obtained from finite element analysis and provided a specific algorithm for fast determination of the unknown parameters. In [4] the Digital Image Correlation technique (DIC) was applied to study the fracture properties of a commercial NiTi pseudoelastic alloy. The near crack tip displacement field of a single edge specimen was measured and the stress intensity factor (SIF) was estimated by using a proper fitting procedure based on the Williams series expansion. The effects of higher-order terms in the Williams expansion (*T*-stress) were analysed for different sizes of the crack tip fitting region and the results were compared with analytical predictions.
The aim of the contribution is to verify the stress fields near the crack tip reconstructed based on the displacement data obtained experimentally via comparison with the stress field approximations derived for the normalized compact tension specimen from hybrid elements [5]. The displacement field in the vicinity of crack tip, which is necessary for the subsequent analysis, is measured in a compact tension (CT) specimen made from Al 2024-T351. Crack tip stress tensor components are expressed using the linear elastic fracture mechanics (LEFM) theory in this work, more precisely via its multi-parameter formulation, i.e. by Williams power series (WPS). Determination of coefficients of terms of this series is performed using a least squares-based regression technique known as over deterministic method (ODM) for which displacements data obtained experimentally via optical measurements are taken as inputs.
Note that this work is only a part of the extensive ongoing research of the authors on the application of this multi-parameter approach in more advanced fracture mechanics tasks.

EXPERIMENTAL STUDY

Material Al 2024 and method of measurement

Experiments were conducted on a CT specimens which was extracted and machined in T-L direction (crack propagation along rolling direction) from a Al 2024-T351 according to ASTM E-647 [6]. Fig. 1 illustrates the specimen geometry and dimensions. The mechanical properties of the material are summarized in Tab. 1. Cyclic loading was applied then with a 100 kN Instron servo hydraulic testing machine. The specimen was pre-cracked under mode I load for 120,000 cycles at a frequency of 10 Hz, a load ratio R = 0.1. Small scale yielding conditions were met in all tests.

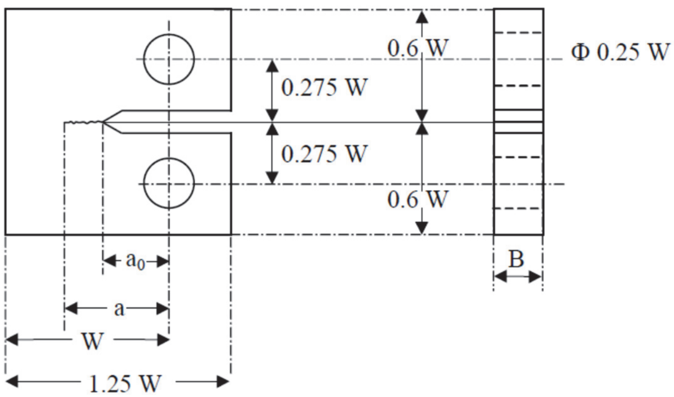


Figure 1: Geometry of the compact tension (CT) specimen in accordance with ASTM standard [6]. W=48 mm, B=24 mm.

Young's modulus	Yield stress	UTS	Elongation at break
73 GPa	325 MPa	470 MPa	20 %

Table 1: Mechanical properties of Al 2024-T351 aluminium alloy.

Method of measurement: Digital Image Correlation (DIC)

DIC was used to investigate the crack tip behaviour of the CT specimen for two different crack lengths and three load levels (see Tab. 2). DIC provides full-field displacement information by comparing images taken before and after straining the body. Each image is divided into smaller regions or interrogation windows. The cross-correlation product [7] is used to measure the similarity between interrogation windows before and after straining the body in study:

$$c(u,v) = \sum_{x=-\frac{N}{2}}^{x=\frac{N}{2}} \sum_{y=-\frac{N}{2}}^{y=\frac{N}{2}} I_A(x,y) \cdot I_B(x+u,y,v) \tag{1}$$

where c is the cross-correlation product which is a function of u and v, the displacement vectors joining the centres of the two regions of interest along directions x and y respectively, I_A and I_B are the intensity distribution of the two digital images before and after straining the sample, respectively, and N is the number of interrogation windows into which the digital images were divided. The maximum value of the cross-correlation product (Eq. 1) is the probable displacement vector for the centre of the each interrogation window in I_A. The camera was placed horizontally so that the positive x coordinate matched the crack growing direction and the y coordinate matched the crack opening direction [8]. This improves and makes the post-processing of the results easier under conditions leading to non-horizontal cracks [9]. DIC requires the surface to have a random pattern so that each interrogation window is unique in each image and can be located easily in the same image after it has undergone some deformation or rigid body movement. In this work, this pattern with random grey intensity distribution was obtained by scratching the sample surface with abrasive SiC sand papers grades 240, 380 and 800 [10], [11] for better imaging of the crack tip [12]. An 8-bit 2452x2052 pixels CCD camera with the maximum frame rate of 12 was used for taking images and recommendations from previous analyses were followed [13]. The experimental setup was similar to the one used previously [14]. DIC generated a pair of matrices, u and v, with displacement values that were combined with an analytical model to infer fracture mechanics parameters (see next section).

MULTI-PARAMETER FRACTURE MECHANICS

Crack tip fields for mode I fracture problem

Williams [15] derived that the crack tip stress and displacement distribution can be expressed by means of a power series. Assuming a plane crack with traction-free faces in a homogeneous linear-elastic isotropic material subjected to arbitrary remote mode I loading, the stress/displacement field around the crack tip obtained by the Williams eigenfunction expansion can be expressed as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} A_n \cdot \begin{Bmatrix} \left[2+(-1)^n+\frac{n}{2}\right]\cos\left(\frac{n}{2}-1\right)\theta - \left(\frac{n}{2}-1\right)\cos\left(\frac{n}{2}-3\right)\theta \\ \left[2-(-1)^n-\frac{n}{2}\right]\cos\left(\frac{n}{2}-1\right)\theta + \left(\frac{n}{2}-1\right)\cos\left(\frac{n}{2}-3\right)\theta \\ -\left[(-1)^n+\frac{n}{2}\right]\sin\left(\frac{n}{2}-1\right)\theta + \left(\frac{n}{2}-1\right)\sin\left(\frac{n}{2}-3\right)\theta \end{Bmatrix} \tag{2}$$

Similarly, the displacement vector can be written as:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \sum_{n=1}^{\infty} \frac{r^{n/2}}{2\mu} A_n \cdot \begin{Bmatrix} \left(\kappa+\frac{n}{2}+(-1)^n\right)\cos\frac{n}{2}\theta - \frac{n}{2}\cos\left(\frac{n}{2}-2\right)\theta \\ \left(\kappa-\frac{n}{2}-(-1)^n\right)\sin\frac{n}{2}\theta + \frac{n}{2}\sin\left(\frac{n}{2}-2\right)\theta \end{Bmatrix} \tag{3}$$

In Eq. 2 and 3, r and θ are polar coordinates centred at the crack tip (considering the crack faces coincident with the negative x-axis), μ is shear modulus defined as $\mu = E/2(1+\nu)$, where E and ν are Young's modulus and Poisson's ratio; n represents the index of the term of the power expansion and κ is Kolosov's constant depending on plane stress or plane strain conditions. Coefficients A_n are functions of relative crack length $\alpha = a/W$ and need to be calculated numerically in most cases. The origin of the coordinates was positioned at the crack tip [16] and no crack front curvature was considered [17]. It is worth mentioning that no bulk effects were taken into account in this study [18]. The truncated form of the Williams power series considering N terms of the expansion is commonly used for the stress/displacement field approximation. When the crack is subjected to mixed-mode loads [19], Eq. 2 and 3 also include a shear mode component that allows the shear mode SIF to be estimated [20], [21].

Over-deterministic method (ODM)

Several methods have been derived and suggested for estimation of the coefficients of the higher-order terms of the Williams expansion. Most of those methods (such as hybrid crack element (HCE) method [22], [23], boundary collocation method (BCM) [24] or others) require advanced mathematical procedures and extensive knowledge of special crack elements or FE code. Therefore, the so-called over-deterministic method (ODM) has been chosen for calculation of the coefficients of the higher-order terms in this paper. This method is based on knowledge of the displacement field data (u, v) in a set of k nodes around the crack tip. These values can be obtained either numerically from finite element simulations on the cracked specimen or experimentally via optical measurements (which is the case of this paper). The (u, v) data can be then together with the nodes coordinates (r, θ) put into Eq. 2. Thus, a system of $2k$ equations arises (in 2D) and the only variables are the coefficients A_n. The procedure for determination of the coefficients of the higher-order terms was programmed in a commercial mathematical package Wolfram Mathematica [25]. The ODM is thoroughly described and tested previously [3], [26], [27]. In this study, the displacements of nodes closest to the crack tip (obtained experimentally) were considered for application of the ODM; the total number of nodes in the radius of 1 mm around the crack tip was approximately 60 for each configuration.

Crack tip fields reconstruction

When the coefficients of the higher-order terms of the Williams expansion are known, the crack tip stress and/or displacement field can be reconstructed by means of Eq. 2 and 3, respectively. In this paper, the approximation of the principal stress σ_1 and von Mises stress σ_{HMH} is presented because these stresses are often used in fracture mechanics criteria and thus, they are important for description of the crack behaviour. Following relations were used for calculation of the stress components:

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \tag{4}$$

$$\sigma_{\mathrm{HMH}} = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2} \tag{5}$$

In order to verify the experimental data, the comparison of the crack tip stress distribution is introduced. The coefficients of the Williams expansion determined via the hybrid crack elements for a normalized CT specimen were taken from the literature [5], for mixed mode see [28].

RESULTS AND DISCUSSION

In the first step of the verification process, the stress intensity factor (SIF) values were compared. The values obtained directly from the fracture tests are compared to values calculated from the displacements measured via DIC and to values obtained by Bednář and Knésl by means of the hybrid crack elements [5]. Data from measurements on three cracked specimens are introduced.

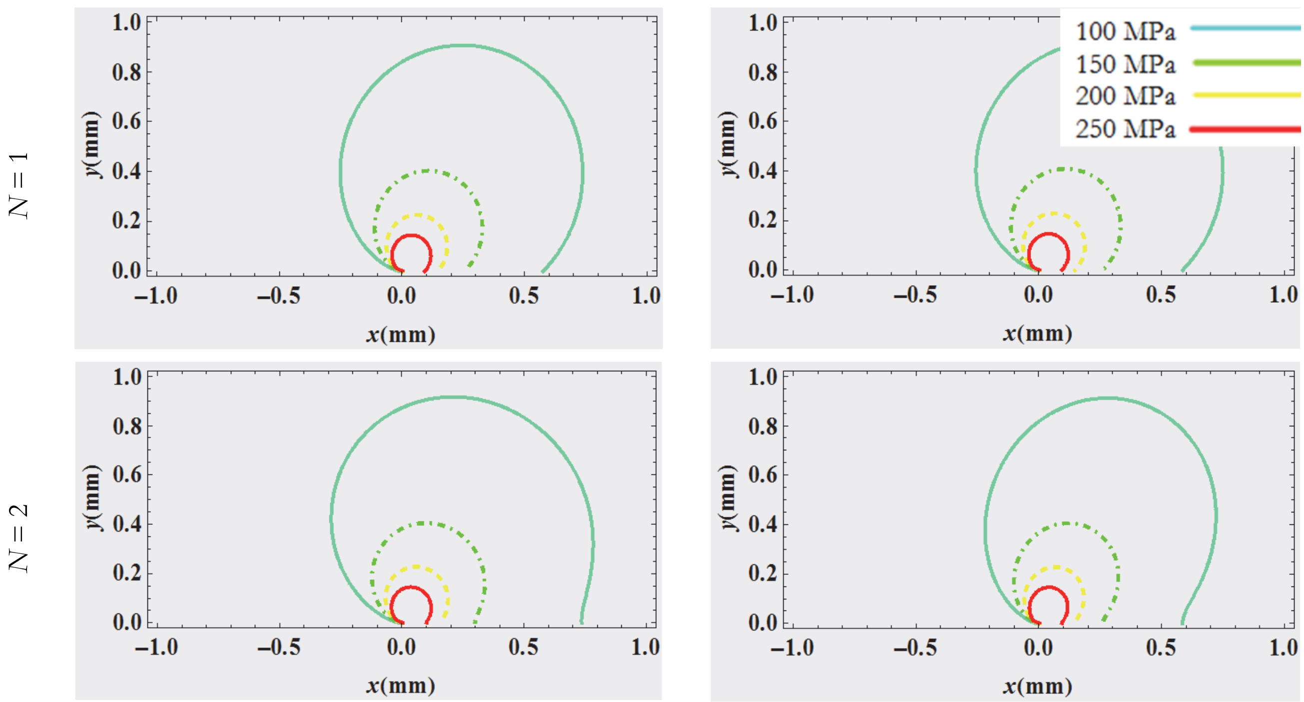
A good agreement can be observed in the results shown in Tab. 2. Thus, further analysis taking into account also the higher-order term (the second one) of the Williams expansion can be performed.

In Fig. 2 to 5 the stress distribution around the crack tip is presented. Particularly, the stress levels with the values of 100, 150, 200 and 250 MPa are plotted for two stress components σ_1 and σ_{HMH}. The values are calculated by means of the Williams expansion taking into account its one or two initial terms and the comparison between the results based on the theoretical values of the coefficients calculated by means of the hybrid elements [5] and the results based on the experimentally measured values of displacements in the specimen. The set of data for two selected specimen configurations are presented, particularly for the specimen YX9 a6_im16 and YX9 a19_im30, see for instance Tab. 2 for more details.

Specimen	Fracture test	Optical measurements (DIC)	Knésl & Bednář [5]
YX9 a6_im16: P = 3.706 kN, a = 22 mm	6.02	6.06	6.01
YX9 a19_im16: P = 3.650 kN, a = 28.62 mm	9.34	9.10	9.45
YX9 a19_im30: P = 7.060 kN, a = 28.62 mm	18.07	18.60	18.28

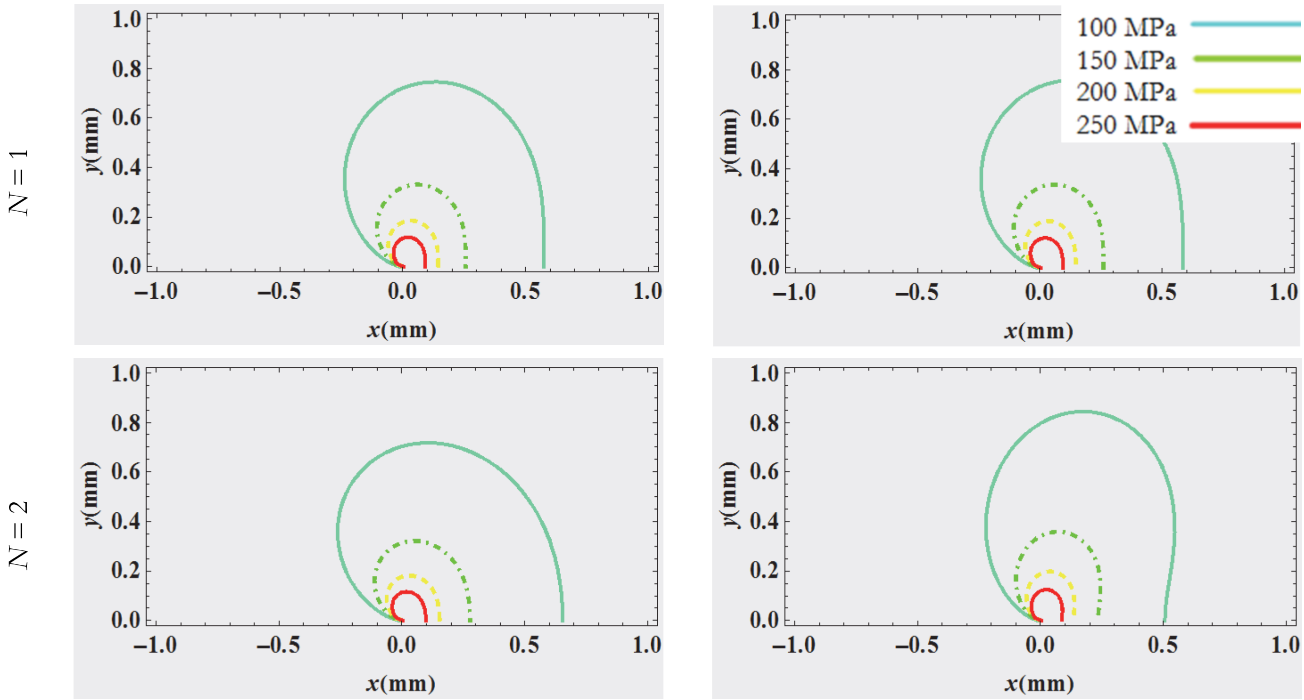
Table 2: Values of the stress intensity factors (SIFs) in MPa√m for three cracked specimens obtained: from fracture tests, via ODM from displacements measured optically by means of DIC, by using hybrid element analysis performed by Knésl and Bednář.

It is clearly seen from the results introduced in Fig. 2 to 5, that the stress levels near the crack tip in the specimen YX9 a19_im30 (with longer crack and larger loading force) are much higher than in the specimen YX9 a6_im16 (with shorter crack and smaller loading force) as it is expected. Another very important conclusion is that the one-parameter fracture mechanics concept as well as the two-parameter fracture mechanics concept can describe rather precisely the stress state near the crack tip. The comparison between the purely theoretical results [5] and results based on the experimental measurement works very well.



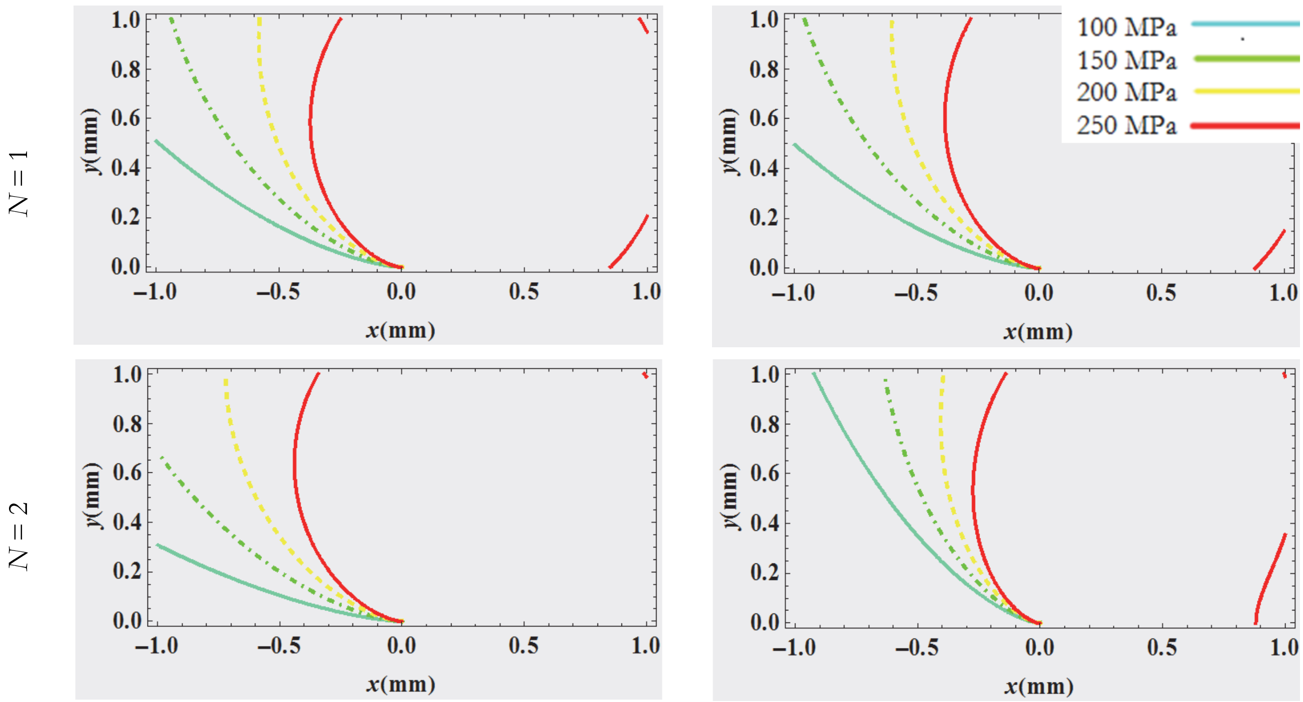
a) Knésl & Bednář [5]   b) experiment + ODM

Figure 2: σ_1 crack tip stress approximation considering one and two initial terms of the Williams expansion on specimen denoted as YX9 a6_im16 (P = 3.706 kN, a = 22 mm): a) stress values calculated from the coefficients determined theoretically [5]; b) stress values calculated from the coefficients obtained by means of the ODM from the experimental measurements.



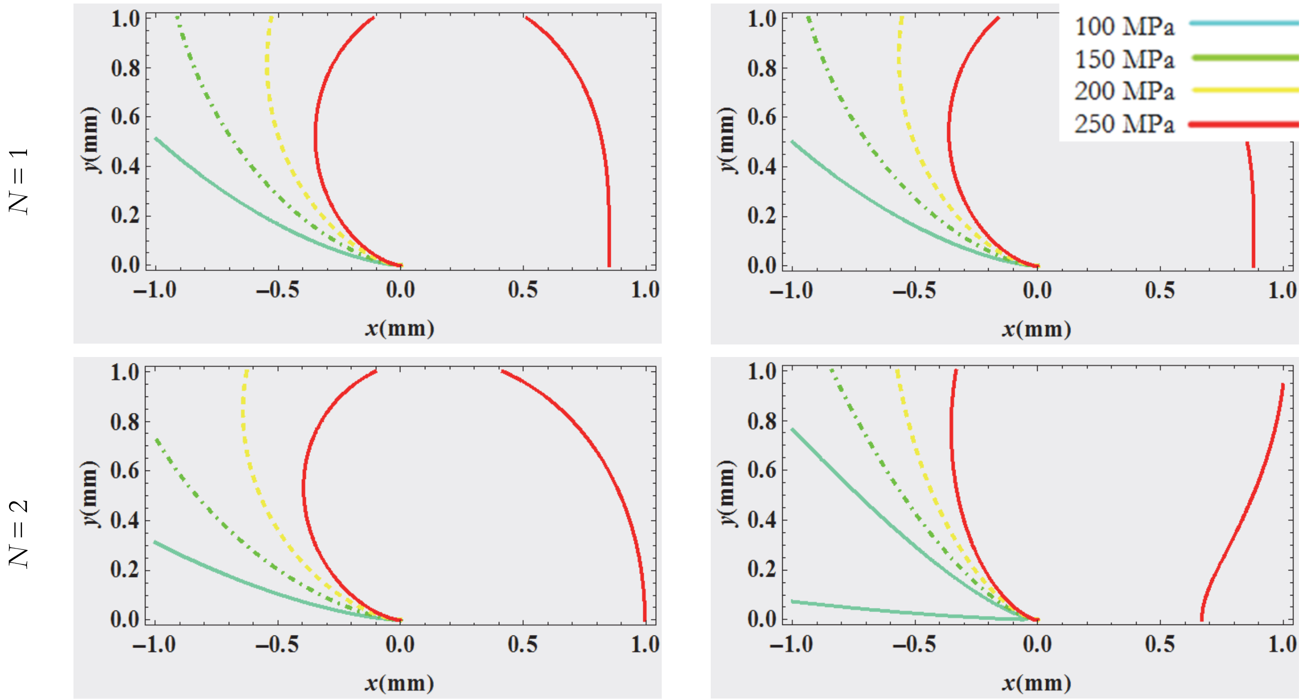
a) Knésl & Bednář [5] b) experiment + ODM

Figure 3: σ_{HMH} crack tip stress approximation considering one and two initial terms of the Williams expansion on specimen denoted as YX9 a6_im16 (P = 3.706 kN, a = 22 mm): a) stress values calculated from the coefficients determined theoretically [5]; b) stress values calculated from the coefficients obtained by means of the ODM from the experimental measurements.



a) Knésl & Bednář [5] b) experiment + ODM

Figure 4: σ_1 crack tip stress approximation considering one and two initial terms of the Williams expansion on specimen denoted as YX9 a19_im30 (P = 7.060 kN, a = 28.62 mm): a) stress values calculated from the coefficients determined theoretically [5]; b) stress values calculated from the coefficients obtained by means of the ODM from the experimental measurements.



a) Knésl & Bednář [5] b) experiment + ODM

Figure 5: σ_{HMH} crack tip stress approximation considering one and two initial terms of the Williams expansion on specimen denoted as YX9 a19_im30 (P = 7.060 kN, a = 28.62 mm): a) stress values calculated from the coefficients determined theoretically [5]; b) stress values calculated from the coefficients obtained by means of the ODM from the experimental measurements.

CONCLUSIONS

In the paper, the crack tip stress distribution is investigated in an Al 2024-T351 body. The standardised CT test was carried out and the displacements data measured optically by means of DIC. These data were used for calculation of the coefficients of the Williams expansion and the stress distribution was reconstructed. The results were compared to the theoretical values of the stress distribution determined from the Williams expansion with the coefficients obtained numerically by means of the hybrid elements. A very good agreement between the both kinds of results was found out. Thus, it can be concluded that the experimental measurement by means of DIC enables precise results to be obtained. These can be used for further fracture mechanics analysis on other types of specimens.

ACKNOWLEDGEMENTS

Financial support from the Czech Science Foundation (projects No. 15-19865Y and 15-07210S), from Junta de Andalucía through Proyectos de Excelencia grant reference TEP-3244 and from the University of Malaga through Lineas Emergentes of Campus de Excelencia International del Mar (CEIMAR) is gratefully acknowledged.
This work is dedicated to the memory of our wonderful colleague, Dr. Vaclav Veselý, who recently passed away.

REFERENCES

[1] Stepanova, L., Roslyakov, P. Multi-parameter description of the crack-tip stress field: Analytic determination of coefficients of crack-tip stress expansions in the vicinity of the crack tips of two finite cracks in an infinite plane medium, Int. J. Solids Struct., 100 (2016) 11–28.
[2] Ramesh, K., Gupta, S., Kelkar, A. A., Evaluation of stress field parameters in fracture mechanics by photoelasticity—

Revisited, Eng. Fract. Mech., 56(1) (1997) 25–45.
[3] Ayatollahi, M. R., Nejati, M., An over-deterministic method for calculation of coefficients of crack tip asymptotic field from finite element analysis, Fatigue Fract. Eng. Mater. Struct., 34(3) (2011) 159–176.
[4] Sgambitterra, E., Lesci, S., Maletta, C., Effects of Higher Order Terms in Fracture Mechanics of Shape Memory Alloys Bydigital Image Correlation, Procedia Eng., 109 (2015) 457–464.
[5] Knésl, Z., Bednář, K., Two-parameter fracture mechanics: determination of parameters and their values (in Czech), IPM AS CR, v. v. i. Brno, 1998.
[6] ASTM E647 Standard test method for measurement of fatigue crack growth rates, (2005).
[7] Clocksin, W. F., da Fonseca, J. Q., Withers, P. J., Torr, P. H. S., Image processing issues in digital strain mapping," in Proceedings of SPIE, Application of Digital Image Processing XXV, 4790 (2002) 384–395.
[8] Mokhtarishirazabad, M., Lopez-Crespo, P., Moreno, B., Lopez-Moreno, A., Zanganeh, M. Optical and analytical investigation of overloads in biaxial fatigue cracks, Int. J. Fatigue, Publ. online, (2017). DOI: 10.1016/j.ijfatigue.2016.12.035.
[9] Lopez-Crespo, P., Moreno, B., Lopez-Moreno, A., Zapatero, J., Study of crack orientation and fatigue life prediction in biaxial fatigue with critical plane models, Eng. Fract. Mech., 136 (2015) 115–130.
[10] Lopez-Crespo, P., Burguete, R. L., Patterson, E. A., Shterenlikht, A., Withers, P. J., Yates, J. R., Study of a crack at a fastener hole by digital image correlation, Exp. Mech., 49 (2009) 551–559.
[11] Lopez-Crespo, P., Shterenlikht, A., Yates, J. R., Patterson, E. A., Withers, P. J., Some experimental observations on crack closure and crack-tip plasticity, Fatigue Fract. Eng. Mater. Struct., 32 (2009) 418–429.
[12] Lopez-Crespo, P., Moreno, B., Lopez-Moreno, A., Zapatero, J., Characterisation of crack-tip fields in biaxial fatigue based on high-magnification image correlation and electro-spray technique, Int. J. Fatigue, 71 (2015) 17–25.
[13] Mokhtarishirazabad, M., Lopez-Crespo, P., Moreno, B., Lopez-Moreno, A., Zanganeh, M., Evaluation of crack-tip fields from DIC data: a parametric study, Int. J. Fatigue, 89 (2016) 11–19.
[14] Lopez-Crespo, P., Garcia-Gonzalez, A., Moreno, B., Lopez-Moreno, A., Zapatero, J., Some observations on short fatigue cracks under biaxial fatigue, Theor. Appl. Fract. Mech., 80 (2015) 96–103.
[15] Williams, M. L., On the stress distribution at the base of a stationary crack, J. Appl. Mech., 24(1) (1957) 109–114.
[16] Zanganeh, M., Lopez-Crespo, P., Tai, Y. H., Yates, J. R., Locating the crack tip using displacement field data: a comparative study, Strain, 49 (2013) 102–115.
[17] Camas, D., Lopez-Crespo, P., Gonzalez-Herrera, A., Moreno, B., Numerical and experimental study of the plastic zone in cracked specimens, Eng. Fract. Mech. Accept. Publ., (2017). DOI: 10.1016/j.engfracmech.2017.02.016
[18] Lopez-Crespo, P., Mostafavi, M., Steuwer, A., Kelleher, J. F., Buslaps, T., Withers, P. J., Characterisation of overloads in fatigue by 2D strain mapping at the surface and in the bulk, Fatigue Fract. Eng. Mater. Struct., 39(8) (2016) 1040–1048.
[19] Lopez-Crespo, P., Pommier, S., Numerical analysis of crack tip plasticity and history effects under mixed mode conditions, J. Solid Mech. Mater. Eng., 2(12) (2008) 1567–1576.
[20] Yoneyama, S., Ogawa, T., Kobayashi, Y., Evaluating mixed-mode stress intensity factors from full-field displacement fields obtained by optical methods, Eng. Fract. Mech., 74 (2007) 1399–1412.
[21] Lopez-Crespo, P., Shterenlikht, A., Patterson, E. A., Withers, P. J., Yates, J. R., The stress intensity of mixed mode cracks determined by digital image correlation, J. Strain Anal. Eng. Des., 43 (2008) 769–780.
[22] Tong, P., Pian, T.H.H., Lasry, S. J., A hybrid-element approach to crack problems in plane elasticity, Int. J. Numer. Methods Eng., 7(3) (1973) 297–308.
[23] Karihaloo, B. L., Xiao, Q. Z., Accurate determination of the coefficients of elastic crack tip asymptotic field by a hybrid crack element with p-adaptivity, Eng. Fract. Mech., 68(15) (2001) 1609–1630.
[24] Xiao, Q. Z., Karihaloo, B. L., Liu, X. Y., Direct determination of SIF and higher order terms of mixed mode cracks by a hybrid crack element, Int. J. Fract., 125(3) (2004) 207–225.
[25] Wolfram Research, Inc., Ed., Wolfram Mathematica Documentation Center. (2007).
[26] Šestáková, L., How to enhance efficiency and accuracy of the over-deterministic method used for determination of the coefficients of the higher-order terms in Williams expansion, Appl. Mech. Mater., 245 (2013) 120–125, 2013. DOI: 10.4028/www.scientific.net/AMM.245.120.
[27] Šestáková, L., Veselý, V., Convergence study on application of the over-deterministic method for determination of near-tip fields in a cracked plate loaded in mixed-mode, Appl. Mech. Mater., 249–250 (2013)76–81. DOI: 10.4028/www.scientific.net/AMM.249-250.76:

[28] Ševčík, M., Hutař, P. Náhlík, L., Seitl, S., The effect of constraint on a crack path, Engineering Failure Analysis, 29 (2013) 83–92, DOI: 10.1016/j.engfailanal.2012.11.011.