ADJUSTABLE FRACTIONAL-ORDER HIGH-PASS FILTER WITH TRANSCONDUCTANCE AMPLIFIERS

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Abstract: This paper deals with proposal of a fractional-order high-pass filter with electronically tunable pole frequency and its order. The filter is designed using operational transconductance amplifiers (OTAs), current follower (CF) and adjustable current amplifiers (ACAs). Function of proposed filter was verified using simulations in PSpice simulator. The circuit features were verified in simulations for three values of order of the filter and four values of cut-off frequency. In this paper, possibility to tune the cut-off frequency using the values of passive components is also mentioned.

Keywords: current mode, electronically tunable, fractional-order, high-pass filter

1 INTRODUCTION

Fractional-order circuits are currently of great interest, therefore the researchers aim their attention also to this matter [1]-[6]. Many designers are interested about design of the fractional-order filters [1]-[4]. Besides design of the fractional-order circuits, the research also deals with design of a fractional-order elements [5], [6]. Fractional-order circuits can be used for electrotechnics, accurate control technology and also for biology [5].

The filter structures are characterized by a transfer function and an order of the filter. The order of the filter determines the slope of attenuation of given transfer function. The slope of attenuation is desribed by $20 \cdot n$ dB/dec, where n is an integer order. The slope of attenuation of the fractional-order filter is described by: $20 \cdot (n+\alpha)$ dB/dec, where α is a number in range $0 < \alpha < 1$ and n is an integer order ($n \geq 1$) [2].

There are two basic methods how to propose a fractional-order filter. The first method is based on a design of a fractional-order passive element. This element can be created by the approximation using a passive RC network [5]. The second method is an approximation of the fractional-order transfer function by an integer-order function. The transfer function contains a fractional-order operator s^{α} which is approximated by an integer-order function [1]-[4]. For the realization of the approximated fractional-order transfer function, the FLF (Follow the Leader Feedback) [1], [4] topology is used, for example. The advantage of this method is the possibility to use available active elements with controllable parameters. Then a proposed fractional-order filter can have an electronic tunable order or a cut-off frequency [4], for instance.

This paper deals with the proposal of the Butterworth fractional-order high-pass filter operating in the current mode. Apart from the design of the fractional-order high-pass filter [1], [2], a fractional-order low-pass filter proposal is most frequently used [1]-[5]. The fractional-order circuits are often proposed in the voltage mode [3], however, fractional-order filters working in the current mode can be also proposed [1], [4].

2 PROPOSAL OF FRACTIONAL-ORDER FILTER

This chapter describes the design of fractional-order high-pass Butterworth filter having order (1+α). General equations and basic topology of the proposed filter are also included.

The basic transfer function [2] of the fractional-order high-pass filter can be given by:

$$K_{1+\alpha}^{\mathrm{HP}}(\boldsymbol{s}) = \frac{k_1}{\frac{1}{\boldsymbol{s}^{\alpha+1}} + \frac{k_2}{\boldsymbol{s}^{\alpha}} + k_3}, \tag{1}$$

where $k_1 = 1$, $k_2 = 1.0683\alpha^2 + 0.161\alpha + 0.3324$ and $k_3 = 0.2917\alpha + 0.71216$. The coefficients k_1, k_2 and k_3 are used for shaping the pass-band region together with the desired fractional-order slope of attenuation of the stop-band [2]. The Butterworth approximation was selected to obtain a maximally flat pass-band response. Parameter s^{α} is fractional-order Laplacian operator which is described in [2], [3]. The following transfer function of fractional-order high-pass filter was created by using function in (1) and s^{α} approximation from [2]:

$$K_{1+\alpha}^{\mathrm{HP}}(\boldsymbol{s}) \cong \frac{k_1}{a_0} \cdot \frac{\boldsymbol{s}^3 a_0 + \boldsymbol{s}^2 a_1 + \boldsymbol{s} a_2}{\boldsymbol{s}^3 + \boldsymbol{s}^2 b_2 + \boldsymbol{s} b_1 + b_0}, \tag{2}$$

where $a_0 = \alpha^2 + 3\alpha + 2$, $a_1 = 8 - 2\alpha^2$, $a_0 = \alpha^2 - 3\alpha + 2$, $b_2 = (a_1(k_3 + k_2) + a_2)/(a_2 k_3 + a_0 k_2)$, $b_1 = (a_0 k_3 + a_2 k_2 + a_1)/(a_2 k_3 + a_0 k_2)$ and $b_0 = a_0/(a_2 k_3 + a_0 k_2)$. The transfer function given in (2) was realized using FLF topology which is shown in Fig. 1. The circuit topology consists of three integrators, three feedback and three forward paths. The transfer function of this block topology is:

$$K(\boldsymbol{s}) = \frac{\boldsymbol{I}_{\mathrm{OUT}}}{\boldsymbol{I}_{\mathrm{IN}}} = \frac{\boldsymbol{s}^3 G_1 + \boldsymbol{s}^2 \frac{G_2}{\tau_1} + \boldsymbol{s} \frac{G_3}{\tau_1 \tau_2}}{\boldsymbol{s}^3 + \boldsymbol{s}^2 \frac{1}{\tau_1} + \boldsymbol{s} \frac{1}{\tau_1 \tau_2} + \frac{1}{\tau_1 \tau_2 \tau_3}}. \tag{3}$$

Comparing particular polynomials from equations (2) and (3) can be determined G_x and τ_x parameters (It is obvious that parameter G_1 is equal $k_1 = 1$ for Butterworth approximation). Values of these parameters are important in order to calculate specific current gains and transconductances of the designed filter.

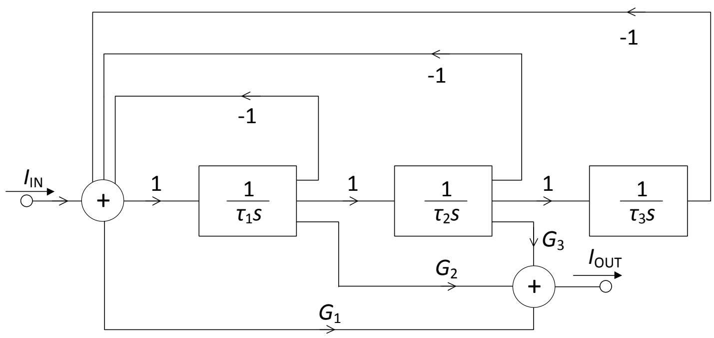

Figure 1: Block diagram of structure used for approximation of a fractional-order high-pass filter

The circuit structure of the proposed filter is shown in Fig. 2. This structure has been designed according to the schema which is illustrated in Fig. 1. The filter structure consists of three OTAs (Operational Transconductance Amplifiers), two ACAs (Adjustable Current Amplifiers), one auxiliary DO-CF (Dual–Output Current Follower) and three capacitors. OTA is described by following equation: $i_{\mathrm{OUT+}} = -i_{\mathrm{OUT-}} = g_m \cdot (u_{\mathrm{IN+}} - u_{\mathrm{IN-}})$, where parameter g_m is electronically controllable transconductance [7]. Current transfer of ACA is described by following relation: $i_{\mathrm{OUT}} = B \cdot i_{\mathrm{IN}}$, where B is current gain [8]. The last active element is DO–CF, which is described by equation: $i_{\mathrm{OUT}} = i_{\mathrm{IN}}$ [8].

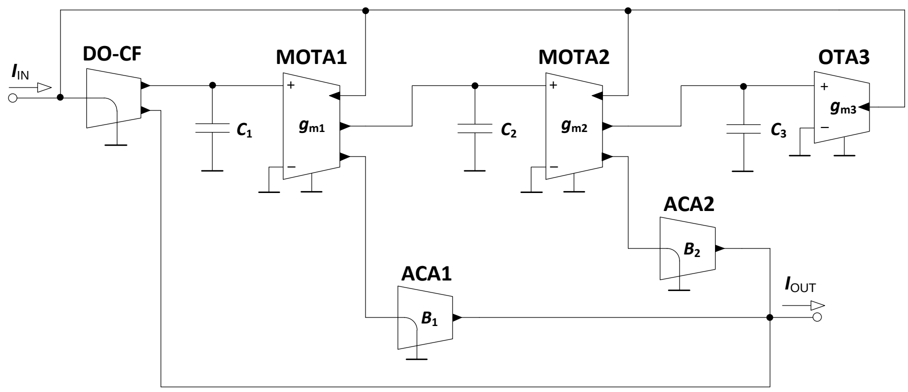

Figure 2: Circuit structure of the designed fractional-order high-pass filter

The transfer function of the circuit from Fig. 2 is:

$$K(s)=\frac{I_{\mathrm{OUT}}}{I_{\mathrm{IN}}}=\frac{s^3+s^2\frac{g_{\mathrm{m1}}B_1}{C_1}+s\frac{g_{\mathrm{m1}}g_{\mathrm{m2}}B_2}{C_1C_2}}{s^3+s^2\frac{g_{\mathrm{m1}}}{C_1}+s\frac{g_{\mathrm{m1}}g_{\mathrm{m2}}}{C_1C_2}+\frac{g_{\mathrm{m1}}g_{\mathrm{m2}}g_{\mathrm{m3}}}{C_1C_2C_3}}. \tag{4}$$

Values of the current gains, transconductances and passive elements can be calculated by comparing each polynomials in the equation (4) with polynomials in (3).

3 SIMULATION RESULTS

The proposed filter was simulated in PSpice. For simulation of the OTA and the DO-CF elements, a 3^{rd} level model of the UCC (Universal Current Conveyor) was used [9]. The ACA element was implemented by EL2082 chip [10]. Starting values of the simulations were chosen as follows: the cut-off frequency $f_0 = 100$ kHz, the order equals 1.5 and the capacitors $C_1 = C_2 = 470$ pF, $C_3 =$ 4.7 nF. The order and the cut-off frequency of the proposed filter can be controlled electronically by changing values of the parameters of active elements. Adjustability of the order of the filter is independent to control of the cut-off frequency. The order is tuned by the current gains B_1, B_2 and the transconductances g_{m1}, g_{m2}, g_{m3}. The cut-off frequency can be tuned by changing of the values of transconductances g_{m1}, g_{m2} and g_{m3}, when keeping their ratio, in order maintaining the same order of the filter. Values of the parameters for three chosen orders (1.3, 1.5 and 1.7) are summarized in Table 1. The Table 2 summarizes required values of the parameters for selected cut-off frequencies (25, 50, 100 and 200 kHz).

Fig. 3 (a) illustrates the possibility to adjust order of the high-pass filter when the cut-off frequency was 100 kHz. It is obvious that simulations (solid lines) are almost identical with theoretical results (dashed lines). The slopes of attenuation of the theoretical results for selected orders are 26, 30 and 34 dB/dec. Values of the slope of attenuation obtained from simulations are 26.3, 31 and 35.4 dB/dec. The phase responses of the proposed high-pass filter for chosen orders are shown in Fig. 3 (b). Electronic tuning of the cut-off frequency for order 1.5 can be seen in Fig. 3 (c). The theoretical values (dashed lines) of the cut-off frequency are 25, 50, 100 and 200 kHz. Values of the cut-off frequency obtained from simulations (solid lines) are 28.2, 55.6, 106.4 and 208.4 kHz. The next possibility to adjust the cut-off frequency is by changing ratio of the values of capacitors. This can be seen in Fig. 3 (d). Selected values of cut-off frequencies are the same 25, 50, 100 and 200 kHz. To shift the cut-off frequency, values of all capacitors were multiplied by 4 for $f_0 = 25$ kHz and by 2 for $f_0 = 50$ kHz. To shift the cut-off frequency to $f_0 = 200$ kHz, values of the all capacitors were divided by 2. Values of the pole frequency obtained from simulation are: 27.7, 56, 106.4 and 209.9 kHz in this particular case. From magnitude and phase responses it can be seen that the results obtained from simulations are in good agreement with theory. The obvious differences between the simulations and theory can be seen at high and low frequencies. This is mainly caused by parasitic properties and limited frequency range of the active elements (their models).

Table 1: Values of the parameters for three selected orders when $f_0 = 100$ kHz

Order	1.3	1.5	1.7
C_1 [pF]	470		
C_2 [pF]	470		
C_3 [nF]	4.7		
$1/g_{m1}$ [Ω]	894	1028	1158
$1/g_{m2}$ [Ω]	3377	3387	3439
$1/g_{m3}$ [Ω]	1051	840	768
B_1 [-]	0.693	0.607	0.515
B_2 [-]	0.116	0.069	0.033

Table 2: Values of the parameters for four values of cut-off frequency when order is 1.5

f_0 [kHz]	25	50	100	200
C_1 [pF]	470			
C_2 [pF]	470			
C_3 [nF]	4.7			
$1/g_{m1}$ [Ω]	4113	2056	1028	514
$1/g_{m2}$ [Ω]	13549	6774	3387	1694
$1/g_{m3}$ [Ω]	3361	1681	840	420
B_1 [-]	0.607			
B_2 [-]	0.069			

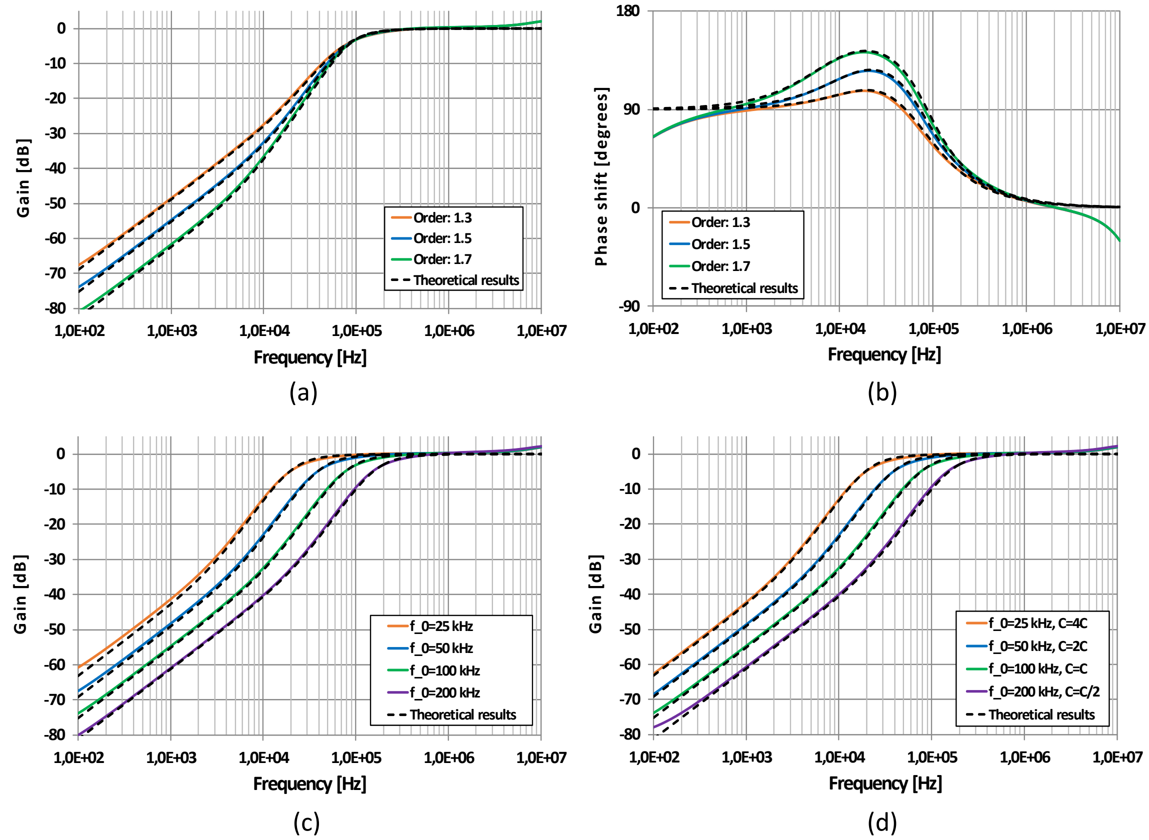

Figure 3: Simulation results of proposed fractional-order high-pass filter: (a) electronic control the order of filter by B_1, B_2, g_{m1}, g_{m2} and g_{m3} when $f_0 = 100$ kHz, (b) phase response for control the order of filter, (c) electronic control the cut-off frequency for order 1.5 by g_{m1}, g_{m2} and g_{m3}, (d) demonstration of possibility to control the cut-off frequency by changing values of capacitors

4 CONCLUSION

This paper describes the proposal of the fractional-order high-pass filter. The possibility of electronic control of the order and cut-off frequency is described. Control of the cut-off frequency by changing values of the capacitors is also mentioned. The function of fractional-order filter was verified using PSpice simulations. The obtained simulation results are compared with theory. It can be stated that the simulations results are very close to the theory for each selected value of the order and cut-off frequency of the proposed filter.

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