

# Requirements of technical standards for the dynamic analysis of the load-bearing structures of footbridges

*Petr Hradil<sup>1,\*</sup>, Vlastislav Salajka<sup>1</sup> and Jiří Kala<sup>1</sup>*

<sup>1</sup>Brno University of Technology, Faculty of Civil Engineering, Department of Structural Mechanics, Veveří 95, Brno, Czech Republic

**Abstract.** The load-bearing structures of footbridges are designed to be slender and feature spans of considerable length. It can be expected that the natural frequencies of such load-bearing structures range from 0.5 Hz to 5.0 Hz. These low natural frequencies are problematic as regards the effects caused by the dynamic component of wind or the movement of persons. Increased acceleration values can lead to the serviceability limit state of structures being exceeded or the heightening of stress which can result in damage to the structure mainly in the area of details prone to fatigue. The contribution deals with the requirements concerning the execution of dynamic analyses which are listed in the relevant technical standards. It will present procedures for the determination of dynamic loading, methods of solving dynamic tasks as well as design criteria enabling delicate bridge structures to be designed correctly.

## 1 Introduction

In the Czech Republic, the evaluation of comfort levels for structures under dynamic loading caused by the movement of pedestrians is primarily covered by the ČSN EN 1990 Standard [12] and other related standards. The standards mainly specify the necessity of meeting comfort criteria and keeping within limit acceleration values. The standards do not deal with the determination of the load which affects the structures.

## 2 Dynamic loading

Loads caused by the movement of pedestrians along structures are interpreted as forces which are a function of position and time. The characteristics of the loading functions are affected by the physiology of the person, their clothes, the stiffness of the structure and other similar factors. Experimental measurements show that the loading function is periodic and is mainly characterized by the pacing frequency. According to performed measurements, standard walking can be described using Gaussian distribution with an

---

\* Corresponding author: [hradil.p@fce.vutbr.cz](mailto:hradil.p@fce.vutbr.cz)

average pacing frequency of 2.0 Hz and a standard deviation of 0.2 Hz. The loading function is usually defined using a Fourier series:

$$F(t) = G_0 + G_1 \cdot \sin(2 \cdot \pi \cdot f_m \cdot t) + \sum_{i=2}^n G_i \cdot \sin(2 \cdot \pi \cdot i \cdot f_m \cdot t - \phi_i) \quad (1)$$

where

$G_0$  the weight of one pedestrian (static value), 700 N

$G_1$  the first harmonic amplitude

$G_i$  the  $i$ -th harmonic amplitude

$f_m$  the pacing frequency

$\phi_i$  the phase displacement of the  $i$ -th harmonic member

In engineering practice, only the first member of the Fourier loading function is used to determine load in calculations, as it does this with sufficient accuracy. The loading function is distributed in the vertical direction,  $F_v$ , and in two horizontal directions, lateral  $F_{ht}$ , and longitudinal  $F_{hl}$ . Function (1) for one freely walking person can be written using the following formulae (2):

$$F_v(t) = G_0 + 0.4 \cdot G_0 \cdot \sin(2 \cdot \pi \cdot f_m \cdot t) \quad (2a)$$

$$F_{ht}(t) = 0.05 \cdot G_0 \cdot \sin\left(2 \cdot \pi \cdot \frac{f_m}{2} \cdot t\right) \quad (2b)$$

$$F_{hl}(t) = 0.2 \cdot G_0 \cdot \sin(2 \cdot \pi \cdot f_m \cdot t) \quad (2c)$$

It needs to be noted that when dealing with a pacing frequency identical to the frequency of transverse loading, half of the value is considered in comparison with the frequencies of vertical and longitudinal loading. This assumption is derived from human walking, during which a transverse frequency results from steps taken alternately with the left and right leg. Footbridge structures are usually exposed to the concurrent movement of several pedestrians. As the movement of any pedestrian can be specified using individual characteristics (weight, pacing frequency, speed of walking, etc.), the loading caused is more or less synchronized. As a result, initial phase displacement of loading arises due to the movement of various pedestrians along the structure of the bridge. In addition, it is also necessary to consider the possibility of modifications to the movement of individual pedestrians. It is very difficult to analyze the stated characteristics of pedestrian movement along structures and implement them in computer software.

It is also very difficult to carry out complete simulations of the effects of the movement of pedestrians. Assumptions can be made and simplifying hypotheses created based on executed studies which assume that loading should be determined via the multiplication of the effects of the movement of one pedestrian using a selected coefficient. Loading from a crowd of pedestrians is therefore determined using two different models. It is a traditional and comprehensive way of dealing with this complex situation. The traditional loading model is used for pedestrians moving independently along bridges where no synchronization is expected. The loading is determined by multiplying the loading caused by one pedestrian by a coefficient,  $k$ . Coefficient  $k$  is determined using formula (3)

$$k = \sqrt{\lambda \cdot T} \quad (3)$$

where

$\lambda$  expresses the number of persons entering the bridge per second,

$T$  is the time the pedestrian takes to cross the bridge.

A comprehensive model of pedestrian loading is able to take the rigidity and natural frequency of the structure into account during calculations. It introduces loading models derived from the number of persons crossing the structure. The equivalent number of persons is added to the calculation as a quantity ( $N_{eq}$ ) for the case of a scattered or compact stream of persons on the structure (4a) or a very dense stream of persons (4b)

$$N_{eq} = 10.8\sqrt{N \cdot \xi}$$

(4a)

$$N_{eq} = 1.85\sqrt{N}$$

(4b)

where  
 $N$  the number of persons on the structure,  
 $\xi$  the damping ratio of the structure.  
Pedestrian-induced dynamic loading used for the calculation of the response is determined for each significant natural frequency. The determination of dynamic loading is also related to the number of persons moving along the structure. A distinction is made between cases when the bridge is crossed by solitary pedestrians, small groups and crowds.

3 Parameters influencing the design, frequency, comfort criteria, damping

The swaying of a footbridge caused by vibration has a negative effect on the movement of pedestrians as they cross it. When designing such structures, it is important to check the parameters that cause vibration, particularly the natural frequencies and natural modes of the vibration, in common the damping value. In the majority of cases, it is sufficient to examine the response of the structure for the first several natural frequencies. This simple method leads to a problem concerning the response of the lowest natural frequencies of the monitored structure. Various rules recommend the examination of the vertical vibration of a structure in a certain frequency range which is given in Table 1, with an emphasis on the characteristics of the movement of persons across the structure.

Table 1. Risk frequency range values noted in literature.

EN 1992	1.60 Hz – 2.40 Hz
EN 1995	0.00 Hz – 5.00 Hz
BS 5400	0.00 Hz – 5.00 Hz
ISO/DIS 10137	1.70 Hz – 2.30 Hz
CEB 209 Bulletin	1.65 Hz – 2.30 Hz

The comfort criteria for the use of a footbridge are usually defined by the owner of the structure. There are three levels comfort level with regard to structural vibration. The first level is the maximum comfort level for pedestrians, who find that the effects of vibration while walking are imperceptible. The second level is one where slight vibration is felt, and the third involves structural vibration that a person can definitely feel, though walking along the structure does not become impossible. The level of comfort is strongly influenced by the users of the bridge and can differ for various groups of inhabitants. A different comfort level is acceptable for children or adults than for older persons or the infirm. It is generally known that there is a difference between the vibrations affecting a structure and

the vibrations actually perceived by pedestrians. Nevertheless, the existing knowledge in this area is still inaccurate and insufficient. This is why, in the case of projects, it is necessary to consider recommended values and use parameters which can be determined using calculations and measurements. Threshold acceleration values are defined which should not be exceeded. Eurocode [2] requires the evaluation of the pedestrian comfort criterion for footbridges. An acceptable level of acceleration is stated as being  $a_{lim} = 0.7 \text{ m/s}^2$  in the vertical direction and  $0.2 \text{ m/s}^2$  in the horizontal direction. The lock-in phenomenon must be mentioned in connection with the effect of pedestrian movement across a structure. When persons begin crossing the bridge, the frequencies of the steps they take exhibit random distribution and differing initial phases. However, the pacing frequency of bridge users eventually synchronizes with the natural frequency of the structure. Amplitudes that should not be exceeded are therefore recommended for vertical (10 mm) and transverse (2 mm) vibration.

The lowering of vibration in built structures is achieved via damping. The most frequent method is via internal damping provided by the construction material itself. Another way of damping structures is through friction between individual structural elements, and their mutual displacement. Such damping is interpreted as a force which limits the movement of the structure, and is referred to as viscous damping. A large proportion of the results of experiments conducted on viscous damping in structures suggest that vibration damping is independent of the natural frequency of the structure but is related to the amplitude of movement. Generally, the critical damping value rises with increasing vibration. Studies have been produced which use damping values ranging from 0.1% to 2.0%. The values used for damping are comprehensively summarised in the CEB bulletin and are listed in Table [2]. It should be mentioned that damping can only be determined via measurements at a previously completed structure.

**Table 2.** Damping of structures.

Type of bridge deck	Damping ratio $\xi$	
	Minimum value	Average value
Reinforced concrete structure	0.8	1.3
Prestressed concrete structure	0.5	1.0
Steel structure	0.2	0.4
Composite structure	0.3	0.6
Wooden structure	1.5	3.0

4 Vibration limits

When acceleration limit values have been exceeded, it is necessary to lower vibration to an acceptable limit. One mechanical means of limiting excessive vibrations is to change the natural frequency of the structure, which involves a radical change in its stiffness or mass, e.g. by adding an auxiliary support. The purpose is to alter the natural frequency of the structure so that it is not near the human pacing frequency. One method of reducing excessive structural vibrations that is considered to be reliable is to increase damping using vibration absorbers in places where significant movement of the structure is expected. Dynamic passive absorbers appear to be suitable for the load-bearing structures of

footbridges. These take the form of a mechanical mass-spring-damper set connected to the original structure. A well-designed absorber causes the energy of the vibrating structure to be absorbed by the additional mass of the damper. Passive dynamic absorbers are suitable for damping the movement of structures with low relative damping and one prevailing frequency.

The first step in designing the parameters of such a vibration absorber is the determination of its weight. A good value is taken as being approximately 1/30 to 1/10 of the vibrating mass of the bridge structure. The next stage involves finding the natural frequency and the relative damping value of the absorber. To this end, use is made of formulae (5), (6) and (7), which can be found in the relevant literature, e.g. [1], [7], [8] or [9]

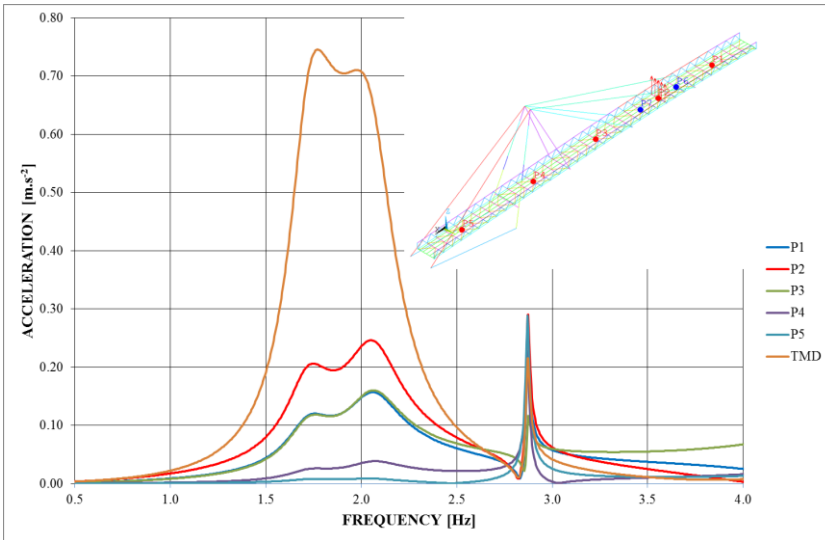
$$f_{TMD} = f_{str} / \left( 1 + \frac{m_{TMD}}{m_{str}} \right) \quad (5)$$

$$\xi_{TMD} = \sqrt{(3 \cdot m_{TMD} / m_{str}) / (8 \cdot (1 + m_{str} / m_{TMD})^3)} \quad (6)$$

$$B_{TMD} = \xi_{TMD} \cdot (2 \cdot m_{TMD} \cdot \omega_{TMD}) \quad (7)$$

where  $f_{TMD}$  is the natural frequency of the absorber,  $m_{TMD}$  is the weight of the absorber,  $f_{str}$  is the natural frequency of the bridge and  $m_{str}$  is the size of the vibrating mass of the bridge for which the absorber is designed. The value  $\xi_{TMD}$  is the damping of the vibration absorber,  $\omega$  is the circular frequency and  $B_{TMD}$  is the damping force proportional to speed.

Formulas (5) to (7) are design formulas for the parameters of the vibration absorber, which must always be specified using dynamic analyses. These analyses are conducted to find optimum parameters. An example of the characteristics of an optimized vibration absorber is shown in Figure 1. It presents the frequency characteristics of the acceleration of a load-bearing structure and a vibration absorber in the monitored frequency.



**Fig. 1.** Characteristics of absorber in a monitored area.

## 5 Conclusion

The contribution describes issues relevant to the calculation of the dynamic response of footbridge structures. Methods of calculating harmonic load imposed by the movement of pedestrians are shown, as well as the parameters affecting the design and evaluation of a structure. These are mainly the natural frequency and relative damping values for the given structure. In the case that permitted or contractually agreed and pertinent structural design criteria are exceeded, vibration absorbers can be required: their function is described. Vibration absorbers are installed in the load-bearing structures of footbridges in order to reduce excessive vibration to an acceptable level.

This paper was created with the financial support of project GACzR 17-23578S “Damage assessment identification for reinforced concrete subjected to extreme loading“

## References

1. H. Bachmann, Lively footbridges – A real challenge. *AFGC and OTUA Footbridge Conference* (Paris, 2002)
2. H. Kreuzinger, Dynamic design strategies for pedestrian and wind actions. *AFGC and OTUA Footbridge Conference* (Paris, 2002)
3. British Standards Institutions, BS 5400, Part 2, Appendix C: Vibration Serviceability Requirements for Foot and Cycle Track Bridges (1978)
4. ČSN EN 1992-2 Eurocode 2 - Design of concrete structures - Concrete bridges - Design and detailing rules (2005)
5. ČSN EN 1995-2 Eurocode 5: Design of timber structures - Part 2: Bridges (2004)
6. J. Stráský, Stress ribbon and cable-supported pedestrian bridges, *Thomas Telford Publishing* (London, 2005)
7. H. Bachman, Vibration problems in structures: *Practical Guidelines*, Birkhauser (Basel, 1995)
8. M. Pirner, O. Fischer, Wind loading of structures, *Information centre of CKAIT* (Prague 2003)
9. J. Kala, P. Hradil, V. Salajka, V. Juttner, Dynamic response of cable stayed steel bridge with tuned mass damper installed - Numerical and experimental approach, *10<sup>th</sup> International Conference on Modern Building Materials, Structures and Techniques* (Vilnius, 2010)
10. V. Salajka, J. Kala, Z. Cada, P. Hradil, Modification of response spectra by probabilistic approach, *Proceedings of the European safety and reliability conference* (Wroclaw, 2004)
11. J. Kralik, J. Kralik, Probability Assessment of Analysis of High-Rise Buildings Seismic Resistance, *Advances in manufacturing science and engineering, Advanced Materials Research* (2013)
12. ČSN EN 1990, Eurocode - Basis of structural design, ed. 2 (2015)