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The use of functional differential equations in the model of the meat market with supply delay

Bobalová Martina[a], Novotná Veronika[b,]*

[a,b] *Brno University of Technology, Kolejni 2906/4, 61200 Brno, Czech Republic*

Abstract

In economic applications, we have to make the assumption that relations between the variables vary with time. One of the possible ways of incorporating the process dynamics into the model is to describe the model by functional equations. The paper is based on the assumption that the balance between the demand and supply can be successfully expressed by a model described by differential equations, even if the goods are supplied with a certain delay.
The equation is solved by modern theory. Theoretical results are illustrated by an example, with concrete results presented in graphical form. The solution is presented by modern computer simulation and the Maple system is used.
The authors come to the conclusion that a delay in the supply of goods can cause an oscillation in the price. On the other hand, it is possible to define conditions under which the solution is monotonous.

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Introduction

One of the typical characteristics of contemporary economic development in the world is the growing interdependence and interconnection of the markets of the national economies and the multinational groups, which is sometimes called the globalization process. Information and the ability to handle information effectively are the source of wealth and power. It is information which plays a decisive role in the success of every individual and

* Corresponding author. Tel.: +420 54114 3718.
E-mail address: NOVOTNA@FBM.VUTBR.CZ.

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organization. The changing world needs information to survive and markets and stock-exchanges are the main driving force of the economy. The entry into the unified market and harmonization of the legal framework have a decisive effect on the economic position of households and companies. To make this effect less costly and socially unacceptable, suitable instruments for examining the effects of the corresponding measures must be used in constructing the strategies of economic development and forecasting. One of the questions most frequently discussed in connection with further economic development and planning is the question of future development of the markets.

A company owner who wants to be successful needs to have information on the environment and the current trends. Such information may protect him in the future against sales problems, sales decline, or profit decreases. The speed of evaluating information, acquisition of new resources, effective communication, and collaboration among people are some of the factors which are essential to a successful economic situation. Agricultural development, for instance, influences to some extent all sectors of industrial production which follow the production of agricultural commodities, and therefore it is essential for users of agricultural products to be able to foresee its future movements. This paper focuses on demonstrating how functional differential equations with delayed argument can be used to model market relations in the poultry meat market in the Czech Republic. The aim of the study is to analyze the behavior of the model from the point of view of oscillatoricity in the case of a change in the input parameter. In the applications section, we show how the model is constructed and how it may be solved in a particular case. The model behavior is presented using computer simulation and the Maple System is utilized in representing the results graphically.

1. Walras´s Model

The idea of market equilibrium, intuitively introduced by Adam Smith, was worked out as a self-contained theory of general equilibrium by Leon Walras. One of Walras´s contributions consisted in the inclusion of the household sector. As a founder of the general equilibrium theory, Walras was followed by other authors, who worked out his theory into a form in which it may be applied to analyzing economic policies.

Leon Walras´s scientific contribution to the development of economics was not fully appreciated until after his death, in particular after World War II, owing to J. R. Hicks and J. Fisher. In the 1930s and 40s, certain economists, such as G. Cassel, F. Hahn, or T. Triffin, inspired by Walras´s theory, connected the model with imperfect competition. J. L. Neumann created a dynamic equilibrium model and a great Walras enthusiast, J. R. Hicks applied Walras´s methodology at the macro-level. After World War II, this concept was further expanded, particularly thanks to G. Debreu and K. J. Arrow. These authors "confirmed the consistency of the general equilibrium theory, including the definition of existence conditions". Under certain strict conditions, they gave a theoretical mathematical proof of Walras´s conjecture which says that adaptation processes will finally lead to a stable equilibrium.

Recently, a number of papers has appeared which study Walrasian dynamics in experimental markets (Plott, 2000; Anderson et al., 2003; Crocket et al., 2011; Hirota et al., 2005). Other studies deal with the economic equilibrium (Anello et al., 2010; Donato et al., 2014; Jofre et al., 2005).

Walras´s model represents a more complex approach to micro-economic analysis as compared with the partial equilibrium methodology represented by the Cambridge School. However, due to its apparent complexity, the model is not often presented in contemporary medium-level textbooks of microeconomics. Various ways of solving the model are studied, for instance, in Danielle (2006) and Anello et al. (2010) books. In the latter, we can find a number of problems which have a background in economics or finance and are formulated in terms of Lebesgue spaces.

An analysis of the dynamics of prices, production and consumption, especially in the case of commodities, can be based on the assumption of the Walrasian theory which says that relative change in the price $p(t)$ in time t is governed by the equation of equilibrium between the demand $D(p(t))$ and $S(p(t))$. The dynamic formalization of this relationship is as follows: $$p' = f(D(p) - S(p)) \qquad (1)$$

Further, we assume that $xf(x) > 0$ pro $x \neq 0$.

In the sequel, the properties of this model will be analyzed using the linear approximation

$$f(x) = k_1 x, k_1 > 0. \tag{2}$$

Also, we assume that

$$D(p(t)) = d_1 + d_2 p(t), d_2 < 0, \tag{3}$$
$$S(p(t)) = s_1 + s_2(p(t)), s_2 > 0,$$

where d_1, d_2, s_1, s_2, k_1 are parameters.

1.1. The model in the case of the delay assumption

We will now accept the idea that the supply side responds to changes in the price with certain delay. This situation can be mathematically formalized as follows:

$$D(p(t)) = d_1 + d_2 p(t), d_2 < 0, \tag{4}$$
$$S(p(t)) = s_1 + s_2 p(t - \Delta), s_2 > 0,$$

where Δ represents the time required to produce a change in the supply, depending on the price development. In what follows, this will be referred to as the delay.

The original equation (1) can then be written as:

$$p'(t) = k_1(d_1 - s_1 + d_2 p(t) - s_2 p(t - \Delta)). \tag{5}$$

2. Constructing the solution

Phenomena which are based on economic reality and described by statistical data can be conveniently modeled by methods based on such branches of mathematics as statistics, numerical methods, operational research, linear and dynamic programming, optimization etc. (David and Křápek., 2013; Fumi et al., 2013).

Around the middle of the 20th century, new mathematical models began to appear which can be used to describe various practical-life situations (Aluf, 2014; Balasubramaniam et all, 2014). This development is closely related to the development of the „Caratheodorian“ theory of differential equations, particularly the increasingly sophisticated theory of functional differential equations. A general theory concerned with the solution of the above-mentioned problem and related problems can be studied, for instance, in the monograph Kiguradze and Půža (2003), which deals with the linear problem. Application problems based on the solution of systems of differential equations with a delay, including a description of how the solution is constructed, can be found, for instance, in Kuchyňková and Maňásek (2006) and in the literature cited therein.

Our construction of the solution uses the Maple system. One of the advantages of this mathematical software is that it allows to find the solution symbolically. To find the solution of our problem we use mathematical procedures which are built into the Maple System and which are designed for solving ordinary differential equations and the above-mentioned modern theory of differential equations with delayed argument.

Let us now consider the mathematical model of the economic process described in the introduction, on the interval $[-\Delta, T]$, in the form:

$$p'(t) = k_1(d_1 - s_1 + d_2 p(t) - s_2(\chi(t - \Delta)p(t - \Delta) + (1 - \chi(t - \Delta))p_h(t - \Delta))), \tag{6}$$
$$p(t) = p_h(t), t \in [-\Delta, 0], \tag{7}$$

where $\chi(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

A natural consequence of the fact that the solution $p(t)$ on the interval $[0, T]$ is a continuous extension of the "historical" function $p_h(t)$ $(t\epsilon[-\Delta, 0])$ is the fact that the initial condition for the solution $p(t)$ of equation (7) has the form

$$p(0) = p_h(0). \tag{8}$$

It follows from Kiguradze and Půža (2003) and from the literature, cited therein, on fully general linear boundary value problems for functional differential equations that:

- the problem (6), (8) has a unique solution
- the solution can be constructed by the method of successive approximations.

Under these assumptions, our problem (6), (8) has a unique solution and this solution is continuously differentiable on the interval $[0, T]$.

To further investigate the properties of the solution of the problem (6), (8) we will use well-known results on the oscillatoricity of the solution of a first-order linear differential equation with a constant delay. From the theory given, for instance in Agarwal (2012), Tinbergen (1931), we can easily deduce that under the assumption

$$\frac{1}{e} < k_1 s_2 \tau e^{-k_1 d_2 \tau} < \frac{\pi}{2} \tag{9}$$

every solution $p(t)$ of the equation (6) oscillates around the equilibrium price p_e and for $t \to \infty$ the solution tends to the equilibrium price p_e. The equilibrium point p_e is asymptotically stable.

3. Poultry meat market

For the past several decades, in particular from the 1960s, meat consumption in the Czech Republic has been increasing. The reason for this increase is that in the past, production of agricultural commodities and products was subsidized by the state. Meat consumption in the Czech Republic reached its peak in 1989 and 1990 – 97.4 and 96.5 kg per person and year, respectively. Afterward there was a decline in meat sales which was due to an increase in prices and price liberalization. A new increase in the consumption of this meat was caused by favorable price relations in comparison with other kinds of meat, the spread of higher finalization products on the domestic market, easy and fast kitchen preparation, and in the 1990s also findings on dietological properties of different kinds of meat. In the last few years, poultry meat has been the cheapest meat on the domestic market in comparison with other kinds of meat.

A significant change in the poultry meat consumption occurred in 2003, when this meat experienced a moderate decline. The next year, however, the consumption increased again to 25.3 kg per person and year. A record increase in poultry meat consumption occurred in 2005 – 26.1 kg per person. For a comparison, consumption in EU countries is around 23 kg. In 2007, there was a decline in poultry meat consumption, which was most probably caused by consumers having concerns about bird flu. At the moment, poultry meat consumption in the Czech Republic ranges between 24 and 25 kg per person and year.

In compiling our model, data from situation and outlook reports of the Ministry of Health of the CR was used. We utilized annual values for the years 2002 – 2014. The corresponding coefficients were obtained regression analysis. The length of the delay is considered constant, it corresponds to the poultry meat producers´ ability to adapt to market changes. The coefficient k_1, which represents sensitivity of the market to changing conditions, is left as an unknown value.

Equation of meat poultry market:

$$p'(t) = k_1(375{,}2 - 166 - 0{,}78p(t) - 1{,}78p(t - 0{,}5)).$$

Equation of historical function:

$$p_h(t) = 0{,}181t^2 + 3{,}4t + 67, t \in [-0{,}5{,}0].$$

From the condition (8) above we are able to conclude that in our case, the solution oscillates at the moment when the coefficient k_1 – market sensitivity – exceeds the value 1.0779. If the coefficient exceeds the value 3.402, then the solution ceases to be asymptotically stable.

The solutions for different values of k_1 are shown in Fig. 1. The value $k_1 = 0.1$ corresponds to the situation when the market sensitivity is very low and the solution tends to an equilibrium state without oscillations. The first graph shows that the solution is monotonous and tends to a constant value. If $k_1 = 1$, the condition under which the solution oscillates is fulfilled and, simultaneously, we are able to observe that the solution slowly tends to an equilibrium state with a decreasing amplitude of the oscillations. The last graph is for $k_1 = 7$. In this graph we are able to see sine waves with an amplitude increasing in time. This means that no equilibrium state can be reached.

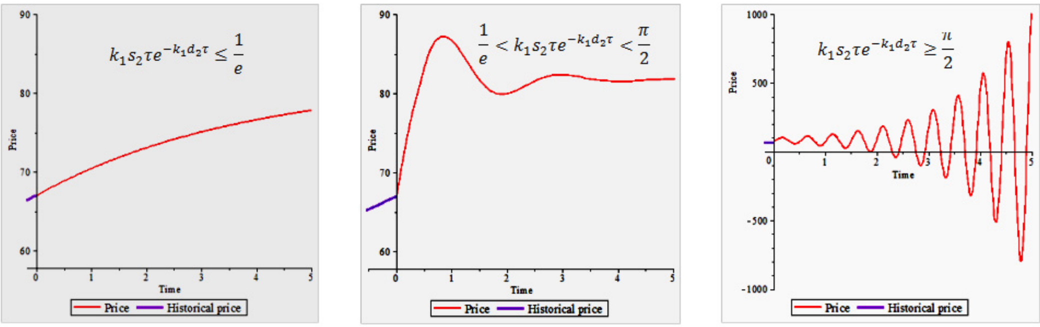

Fig. 1. The solutions for different values of k_1

Conclusions

The tools provided by functional analysis can be successfully used for processing certain type of information in order to maintain or even increase a company´s competitiveness and reduce its risks in the competition struggle. The modeling of economic processes gives us a clear idea of the activities and related aspects of the modeled area. A formal description of the model enforces a clear, brief and graphic catching of reality and very often reveals inconsistent behavior of the modeled process.

This paper presents one of the possible ways of solving the Walras model of market equilibrium in the case where the model is described by an ordinary differential equation with delayed argument. Modern theory is used in solving the equation and conditions under which the solution is oscillatory are analyzed.

Theoretical results are illustrated by an example and concrete results are described graphically. The market under investigation is the poultry meat market in the Czech Republic. As we can see from the results, the behavior of the market in various situations can be simulated by suitable software which permits to improve the quality of the decision-making process.

The model presented in the paper can be successfully expanded to obtain more accurate information on the behavior of the model under investigation. One of the attractive aspects of the modern method of solution used in the paper is that provided suitable software, considerably wider possibilities of analyzing the problem can be obtained. This may prove helpful in further research. Also, a generalization of the model which would admit non-constant delay can be considered.

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